

System Considerations of Pre-filtering and Post-filtering

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- Introduction
- Spectra of Sampled Signals
- Prefiltering
- Postfiltering
- Oversampling Approach
- Conventional Switched-Capacitor Approach
- Examples

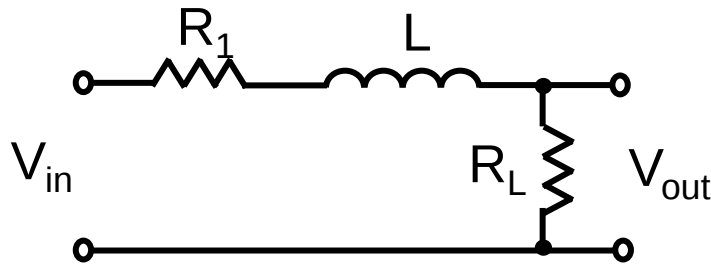
Filters

- Continuous-time filter
 - ◆ RLC passive
 - ◆ Active RC
- Sampled-Data filter
 - ◆ Switched-Capacitor filter
- Digital filter

Continuous-Time Filters

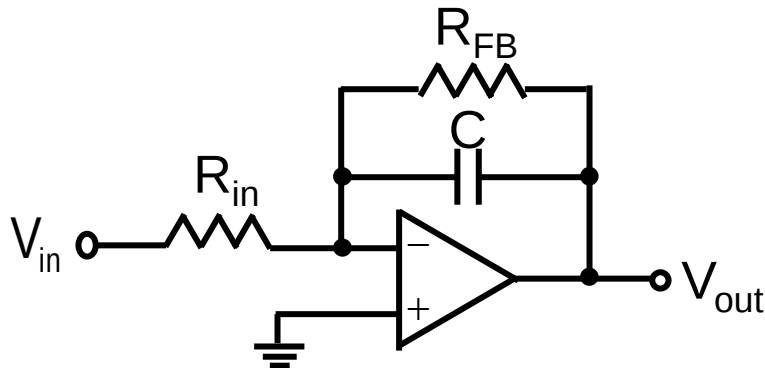
- Example: 1 pole low pass filter

- ◆ Passive



$$\frac{V_{out}}{V_{in}} = \frac{R_L}{R_1 + R_L} \left[\frac{1}{1 + \frac{sL}{R_1 + R_L}} \right]$$

- ◆ Active



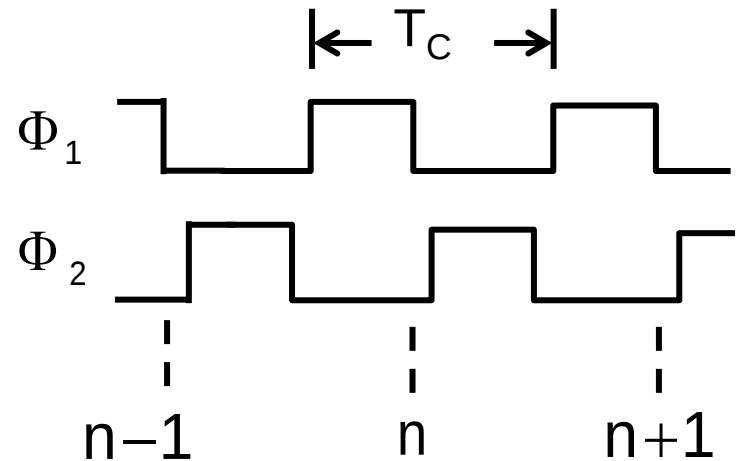
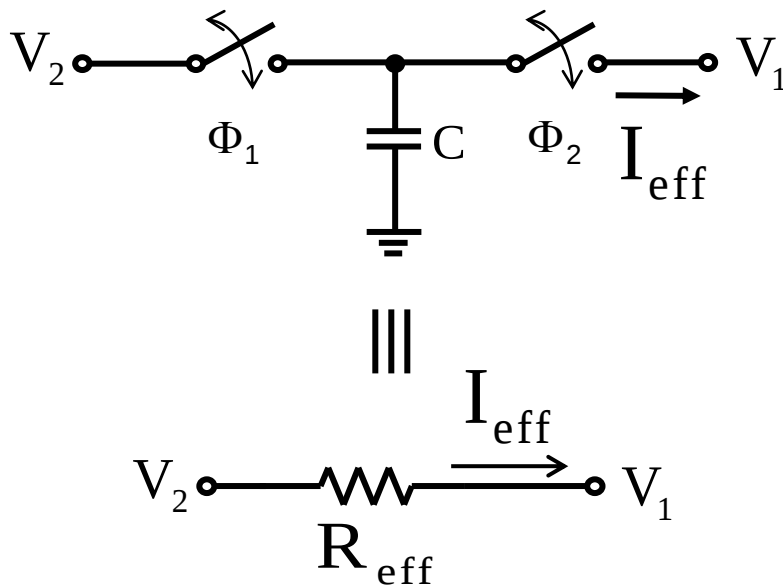
$$\frac{V_{out}}{V_{in}} = -\frac{R_{FB}}{R_{in}} \left[\frac{1}{1 + sR_{FB}C} \right]$$

- ◆ Equivalence conditions of the above

$$R_{in} = R_1, R_{FB} = \frac{R_1 R_L}{R_1 + R_L}, C = \frac{L}{R_1 R_L}$$

Switched-Capacitor Filter (SCF)

- Basic concept



$$\Delta Q = C(V_2 - V_1) \Rightarrow I_{\text{eff}} = \frac{\Delta Q}{T} = \frac{V_2 - V_1}{R_{\text{eff}}} \Rightarrow R_{\text{eff}} = \frac{T_C}{C}$$

SCF (Cont.)

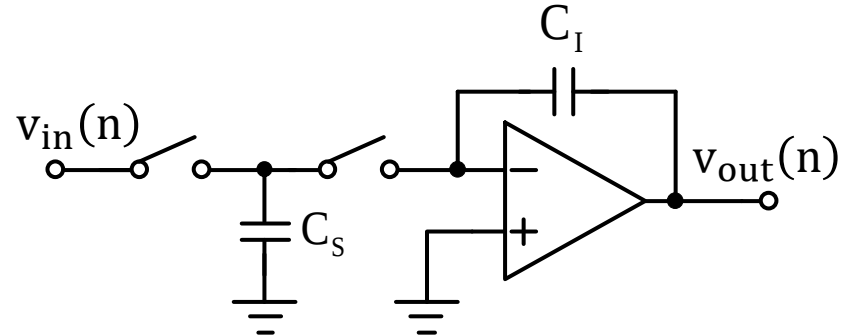
- Example: SC integrator (stray-sensitive)

$$v_{\text{out}}(n) - v_{\text{out}}(n-1) = -\frac{C_S}{C_I} v_{\text{in}}(n-1)$$

$$\Rightarrow v_{\text{out}}(n) = v_{\text{out}}(n-1) - \frac{C_S}{C_I} v_{\text{in}}(n-1)$$

$$\Rightarrow V_{\text{out}}(z) = z^{-1} V_{\text{out}}(z) - \frac{C_S}{C_I} z^{-1} V_{\text{in}}(z)$$

$$\Rightarrow H(z) = \frac{V_{\text{out}}(z)}{V_{\text{in}}(z)} = -\frac{C_S}{C_I} \frac{z^{-1}}{1 - z^{-1}}, \text{ where } z = e^{j\omega T}$$

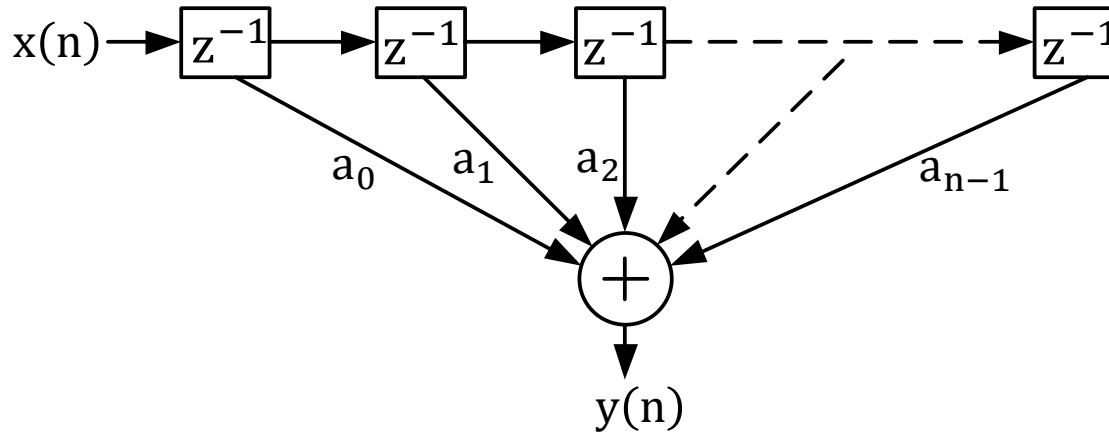


If $\omega T \ll 1$

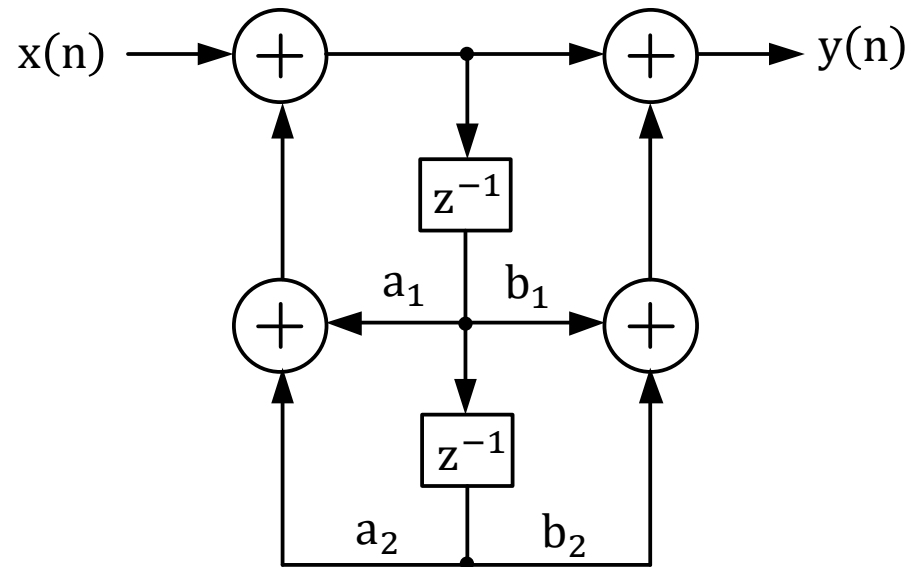
$$\begin{aligned} \lim_{\omega T \ll 1} H[e^{j\omega T}] &= -\frac{C_S}{C_I} \frac{e^{-j\omega T}}{1 - e^{-j\omega T}} = -\frac{C_S}{C_I} \frac{\left[1 - j\omega T + \frac{(j\omega T)^2}{2} - \dots\right]}{j\omega T - \frac{(j\omega T)^2}{2} + \dots} \\ &\approx -\frac{C_S}{C_I} \frac{1}{j\omega T} = -\frac{1}{j\omega \left(\frac{T}{C_S}\right) C_I} = -\frac{1}{j\omega R_{\text{eff}} C_I} \end{aligned}$$

Digital Filter

- FIR (Finite Impulse Response)



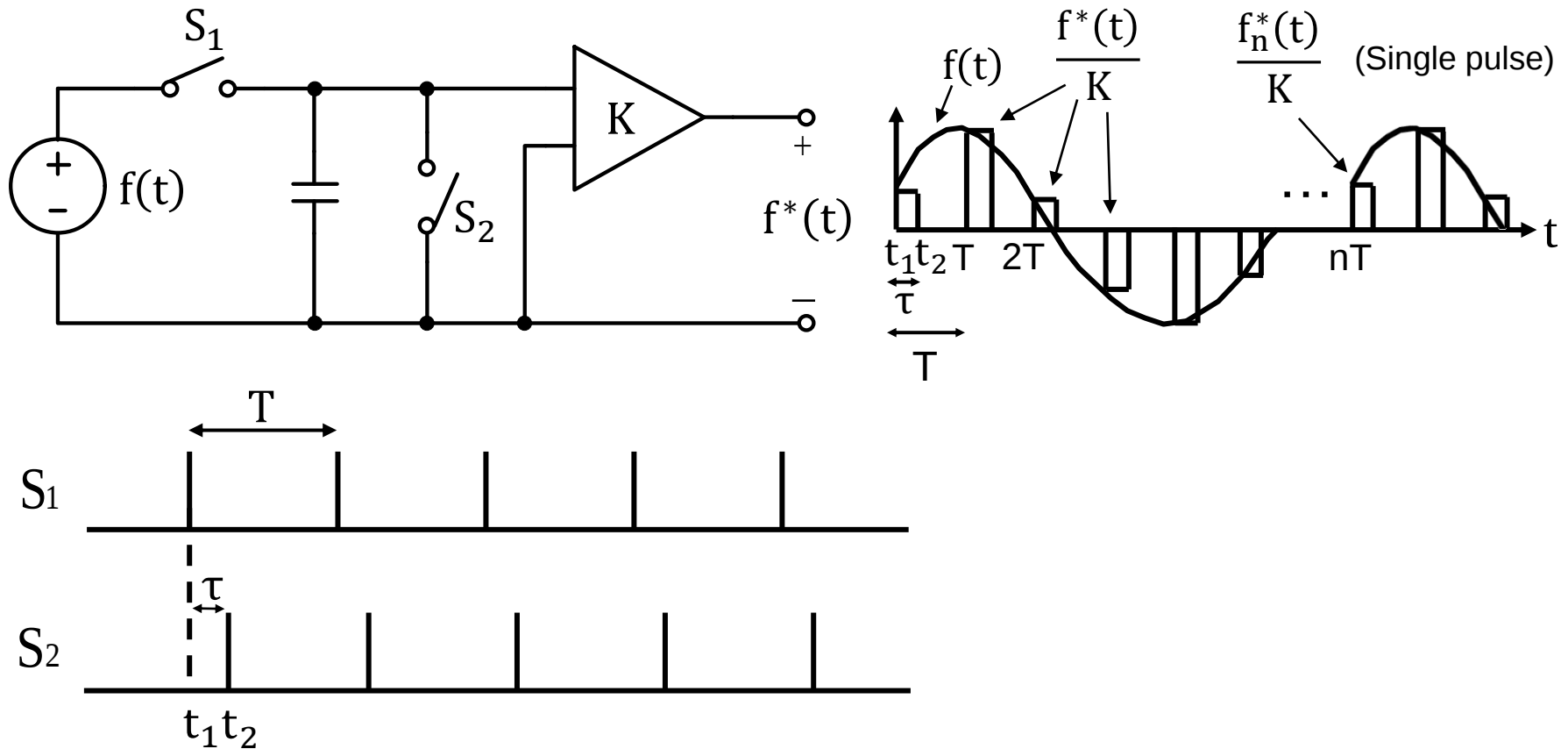
- IIR (Infinite Impulse Response)



- Operations of digital filter

- ◆ Multiply
- ◆ Delay
- ◆ Add

Analog Sampling Circuit



- Original continuous-time signal $f(t)$
- Sampled/Held Signal $f^*(t)$

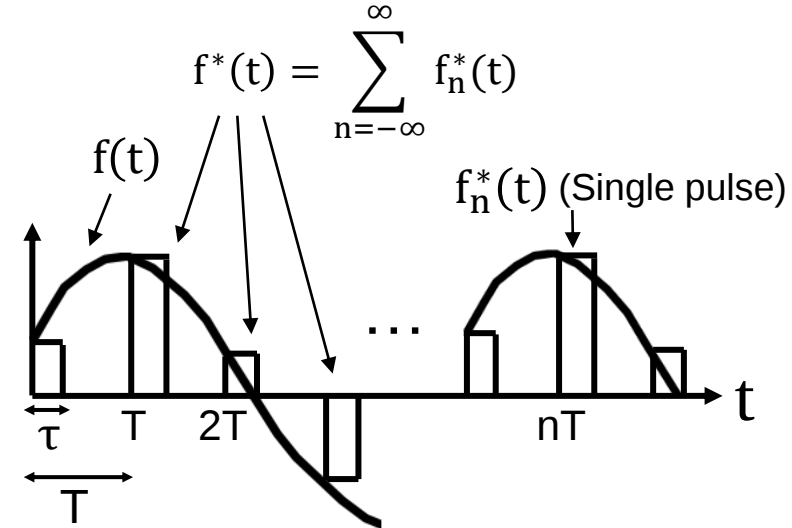
Analog Sampling Circuit (Cont.)

- Mathematical relationship (assume $K=1$)

- Single-pulse signal $f_n^*(t)$

$$\rightarrow f_n^*(t) = \frac{f(nT)}{\tau} [\theta(t - nT) - \theta(t - nT - \tau)]$$

$$\text{where } \theta(t) \equiv \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad (13.3)$$



- Sampled signal for all time

$$\rightarrow f^*(t) = \sum_{n=-\infty}^{\infty} f_n^*(t) = \sum_{n=-\infty}^{\infty} \frac{f(nT)}{\tau} [\theta(t - nT) - \theta(t - nT - \tau)] \quad (13.4)$$

- Laplace transform for $f_n^*(t)$ and $f^*(t)$

$$\rightarrow F_n^*(s) = \frac{1}{\tau} \left(\frac{1 - e^{-s\tau}}{s} \right) f(nT) e^{-snT} \quad (13.5)$$

$$\rightarrow F^*(s) = \frac{1}{\tau} \left(\frac{1 - e^{-s\tau}}{s} \right) \sum_{n=-\infty}^{\infty} f(nT) e^{-snT} \quad (\text{for } \tau \rightarrow 0) = \sum_{n=-\infty}^{\infty} f(nT) e^{-snT} \quad (13.7)$$

Signal Spectra of Zero-Width Samples

- For $\tau \rightarrow 0$ (Zero-width sampling)

$f(t) \longrightarrow F(j\omega)$ Original signal

$f^*(t) \longrightarrow F^*(j\omega)$ Sampled-data signal

- Spectrum of sampled signal

◆ Method1: Replace $s \rightarrow j\omega$ in (13.7)

◆ Method2:

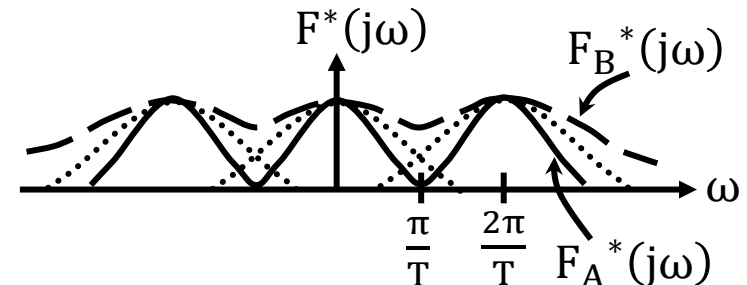
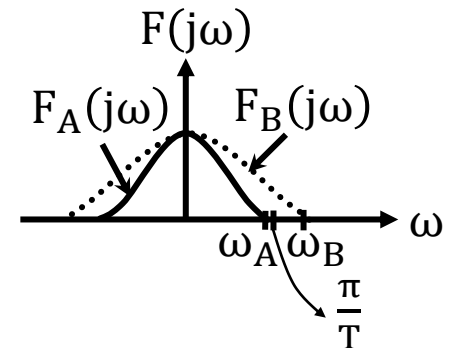
$$f^*(t) = f(t) \cdot s(t) \xrightarrow{\text{Fourier transform}} F^*(j\omega) = \frac{1}{2\pi} F(j\omega) \otimes S(j\omega)$$

Convolution

$$\text{where } s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$S(j\omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$$

$$\Rightarrow F^*(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} F\left(j\omega - jk\frac{2\pi}{T}\right)$$



- Continuous-time signals $f_A(t)$ and $f_B(t)$
- Sampled-Data signals $f_A^*(t)$ and $f_B^*(t)$

Nyquist Theorem

- There is a one-to-one relation between values $F_{A/B}(j\omega)$ and $F_{A/B}^*(j\omega)$
 - ◆ Replicas forming $F_B^*(j\omega)$ overlap
 - It is called aliasing or folding and results in nonlinear distortions

- Low-pass Filter

$$F_A^*(j\omega)H(j\omega) = F_A(j\omega) \text{ where } H(j\omega) = \begin{cases} 1 & ; \left| \omega \leq \frac{\pi}{T} \right| \\ 0 & ; \left| \omega > \frac{\pi}{T} \right| \end{cases}$$

- ◆ The continuous-time signal $f_A(t)$ is recovered
 - ◆ But no such operation can regain $F_B(j\omega)$ from $F_B^*(j\omega)$
- Nyquist first observes this phenomenon
 - ◆ Nyquist Theorem : $f_{\text{sampling}} > 2f_{\text{signal band}}$

Sample/Hold (S/H) Effect

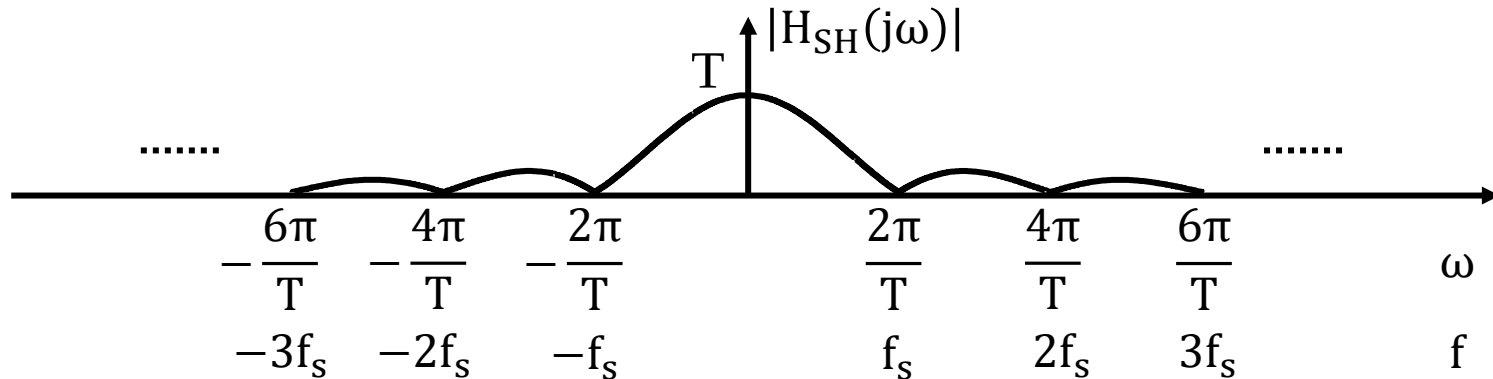
- Due to nonzero-width samples, assume $\tau = T$ and from (13.5)
 $\rightarrow F_{SH}(j\omega) = H_{SH}(j\omega)F^*(j\omega)$, where $F^*(j\omega)$ is spectra of zero-width samples

- Spectrum of S/H response

$$H_{SH}(s) = \frac{1 - e^{-sT}}{s}$$

Let $s \rightarrow j\omega$

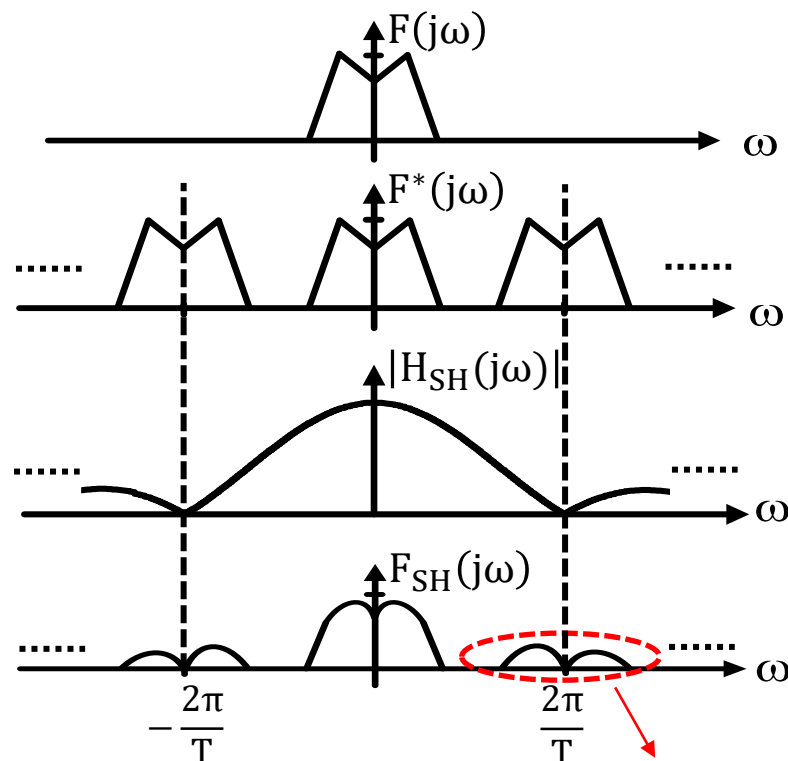
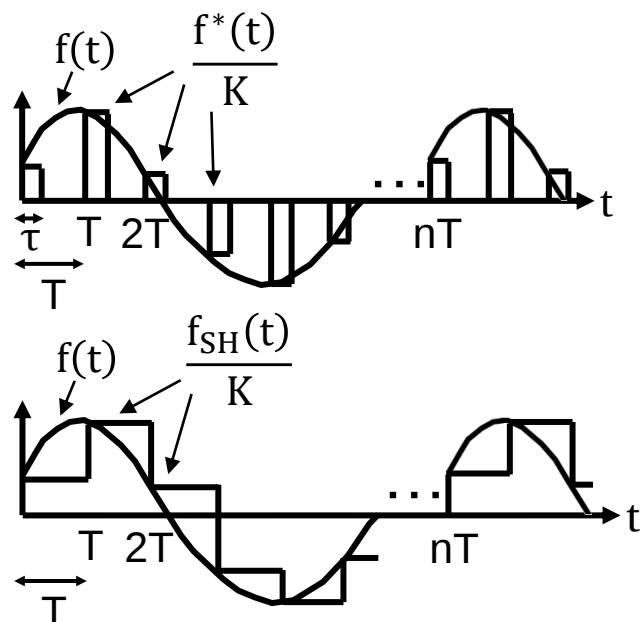
$$H_{SH}(j\omega) = \frac{1 - e^{-j\omega T}}{j\omega} = T e^{-\frac{j\omega T}{2}} \left(\frac{\sin \frac{\omega T}{2}}{\frac{\omega T}{2}} \right) \text{ or } |H_{SH}(f)| = T \frac{|\sin(\frac{\pi f}{f_s})|}{|\frac{\pi f}{f_s}|}$$



- $H_{SH}(j\omega)$ has a linear phase. Its amplitude has sinc response ($\sin X/X$)

Sample/Hold (S/H) Effect (Cont.)

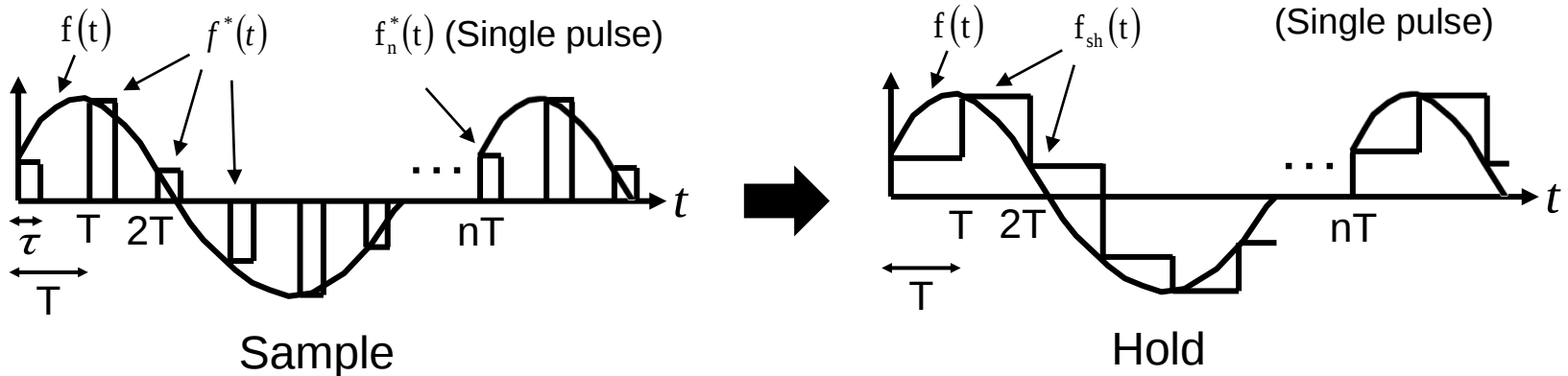
- $F_{SH}(j\omega) = H_{SH}(j\omega)F^*(j\omega)$
- $F^*(j\omega)$ is formed by replicas of $F(j\omega)$
- $F_{SH}(j\omega)$ is replicated and multiplied by the $\sin X/X$ response.



- S/H before an ADC
 - ◆ Allow ADC to have a constant input value during one conversion
 - ◆ Relax the anti-aliasing requirement

Derivation of S/H Function $H_{SH}(s)$

- S/H response (refer to p.9-8)



◆ For sampled signal: $f^*(t) = \sum_{n=-\infty}^{\infty} \frac{f(nT)}{\tau} [\mathcal{G}(t - nT) - \mathcal{G}(t - nT - \tau)]$ (13.4)

- ◆ Assume $\tau = T$ (Nonzero-width sampling)

$\rightarrow f_{sh}(t) = \sum_{n=-\infty}^{\infty} f(nT) [\mathcal{G}(t - nT) - \mathcal{G}(t - nT - T)]$ (13.39)

◆ Laplace transform: $F_{SH}(s) = \frac{1 - e^{-sT}}{s} \sum_{n=-\infty}^{\infty} f(nT) e^{-snT} = \frac{1 - e^{-sT}}{s} F^*(s)$ (13.40)

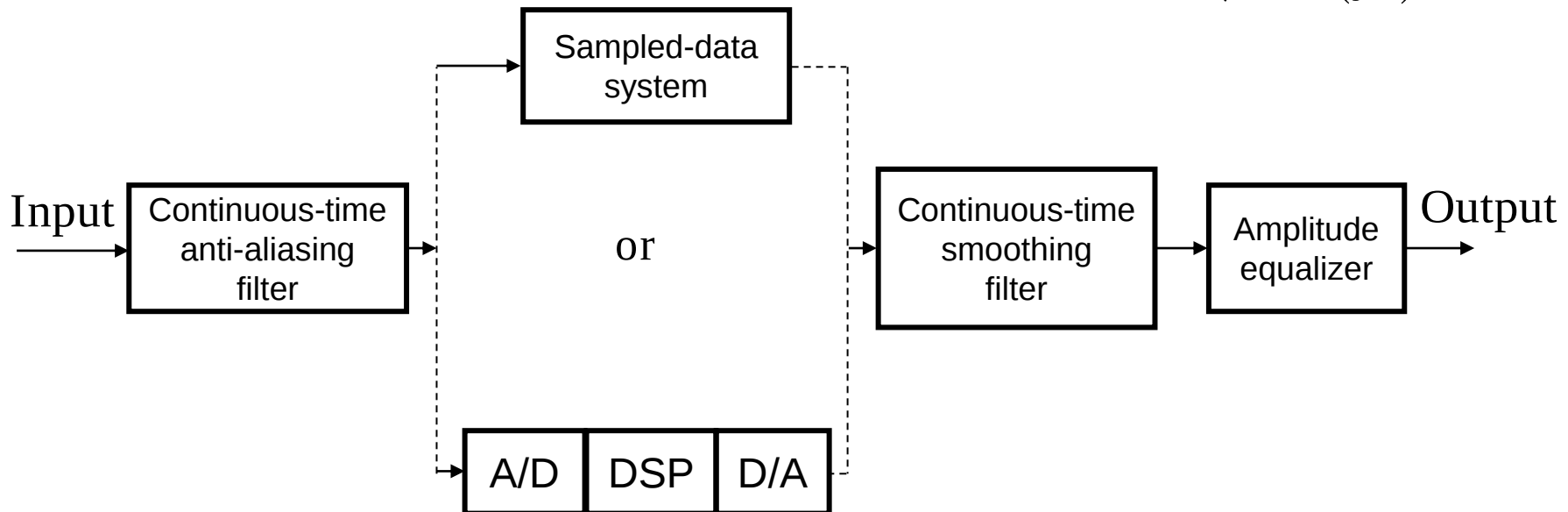
$\rightarrow$ Sample/Hold transfer function, $H_{SH}(s)$, is equal to

$H_{SH}(s) = \frac{1 - e^{-sT}}{s}$ (13.41)

Sampled-Data System with Continuous-time Input & Output Signals

- Distortion due to $H_{SH}(j\omega)$ is linear as opposed to nonlinear aliasing distortion
- $F(j\omega)$ can be recovered from $F_{SH}(j\omega)$ by two steps
 - ◆ Low-pass filter
 - ◆ Amplitude equalizer with a transfer function

$$H_{EQ}(j\omega) = \frac{1}{H_{SH}(j\omega)}$$



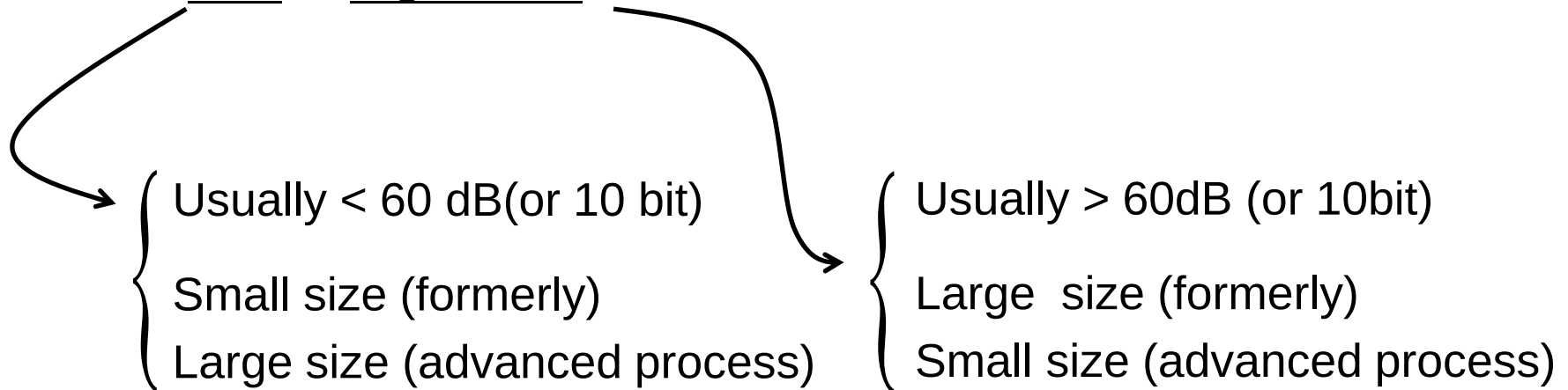
- The anti-aliasing and smoothing filters can be identical lowpass filters, and should ideally have sharp cutoff frequency (except oversampling rate signal processing where decimation and interpolation are used)

Prefiltering

- Nyquist rate
 - ◆ Prefilter = Anti-alias filter (AAF)
 - ◆ Brick wall AAF
- Oversampling rate
 - ◆ Prefilter = Anti-alias filter + Decimation filter
(AAF) (DF)

AAF: Continuous-time filter

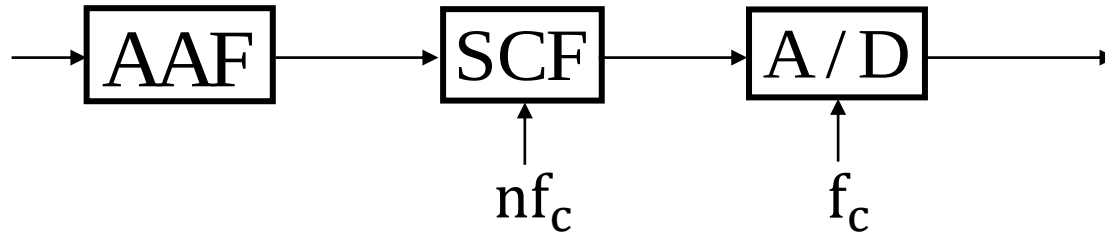
DF: SCF or Digital filter



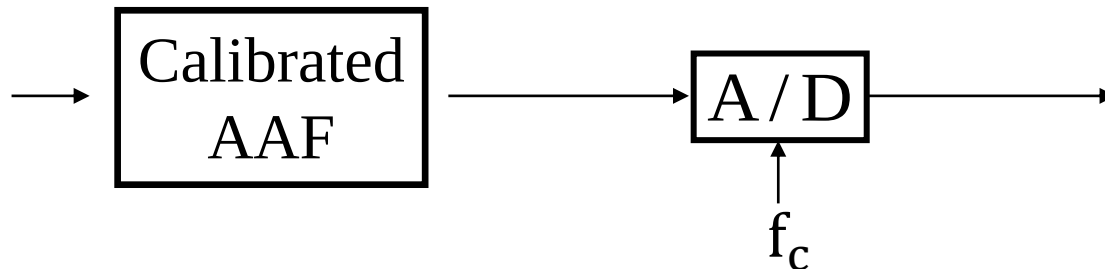
Prefiltering (Cont.)

- Examples: (For Data Acquisition)

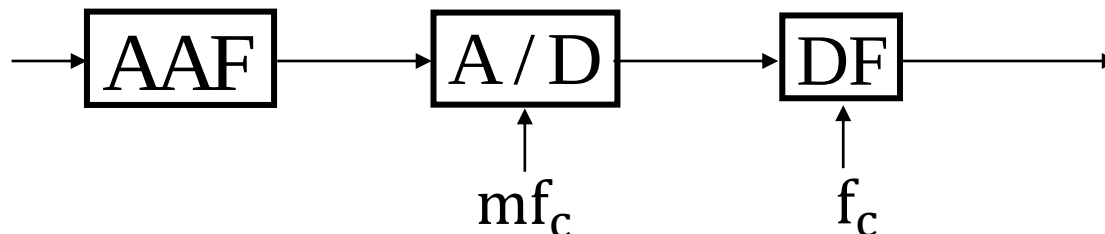
- ◆ Conventional Nyquist-rate A/D converter



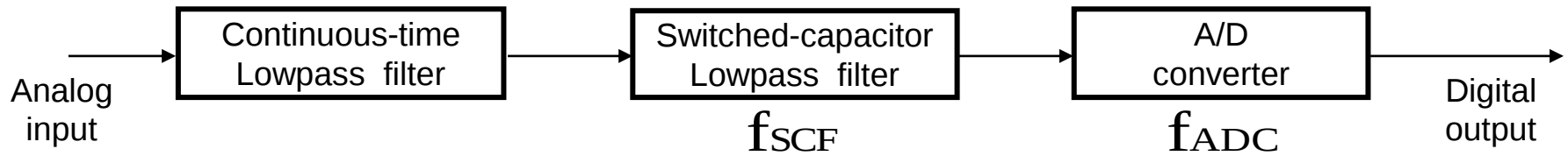
- ◆ Modern high-speed design with Nyquist-rate A/D converter



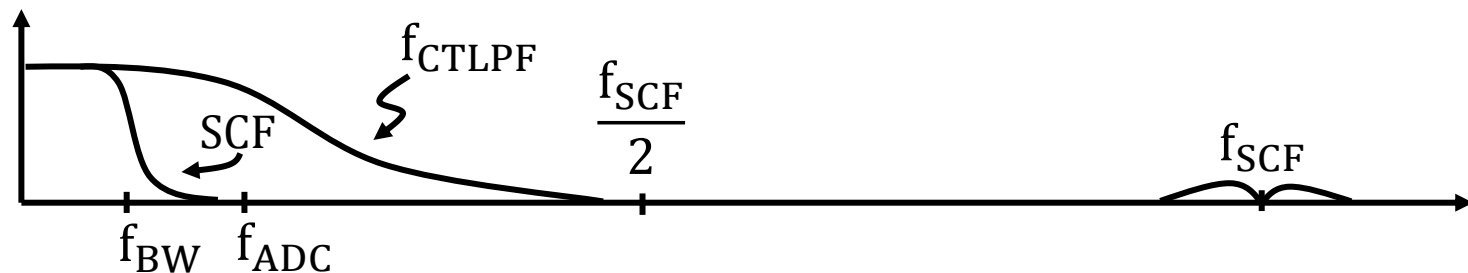
- ◆ Oversampling A/D converter



Prefilter Strategy for Conventional Data Acquisition

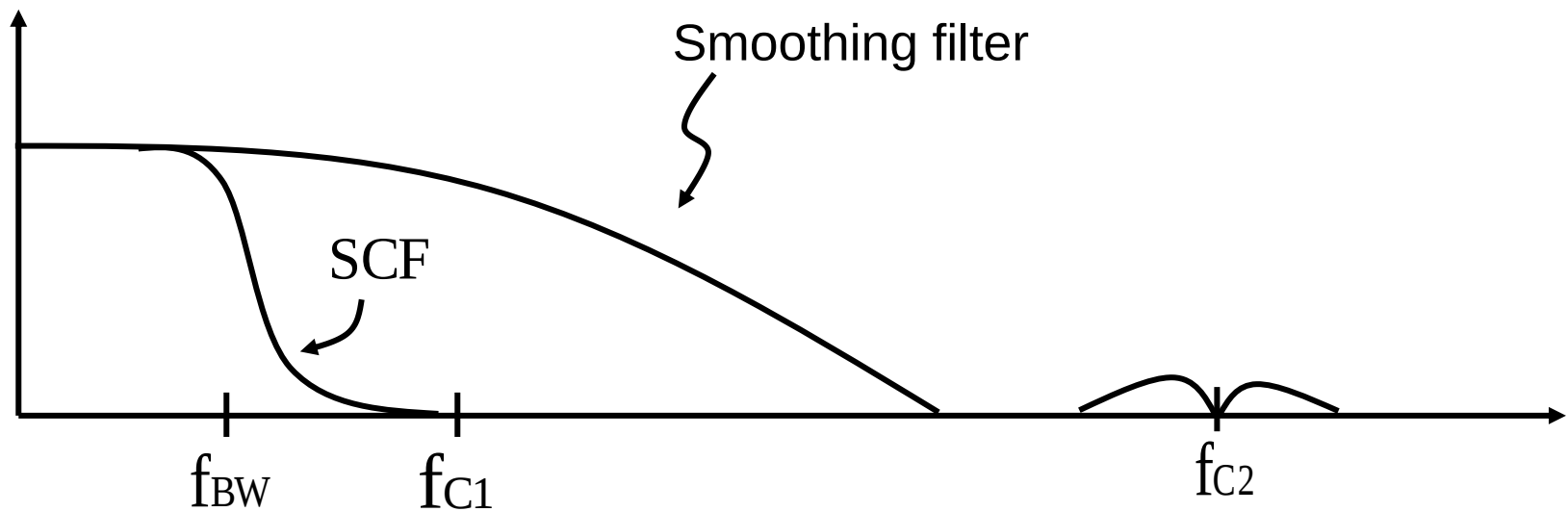
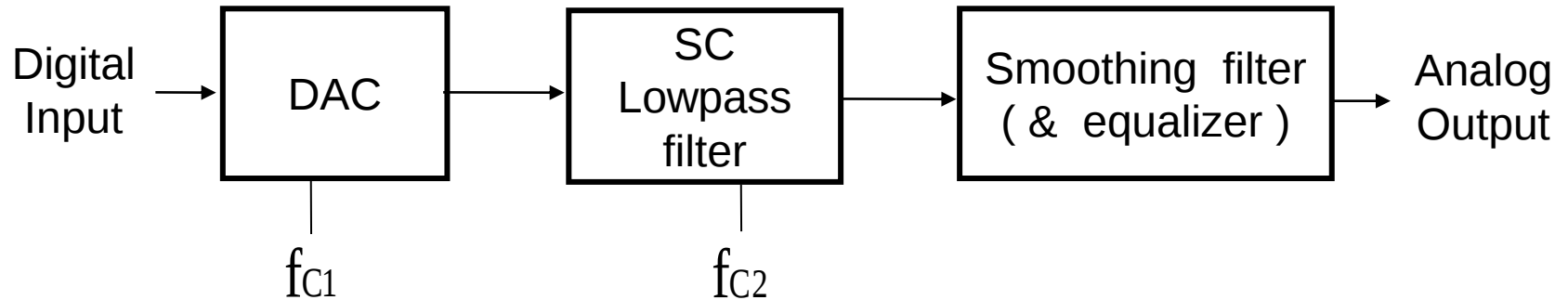


- Provide anti-aliasing for ADC with SCF sampling at a frequency much greater than twice the bandwidth of interest.
- Provide anti-aliasing for SCF by continuous-time low-pass filter (CTLPF) with corner frequency comfortably between $f_{\text{SCF}}/2$ and f_{BW}



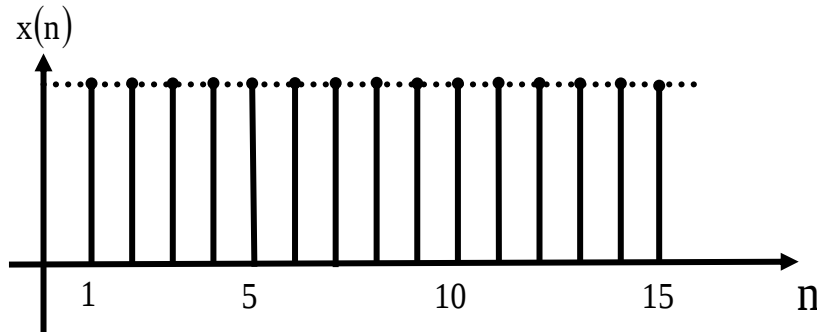
- $f_{\text{SCF}} \gg f_{\text{ADC}} \rightarrow$ SCF performs anti-aliasing filter $\rightarrow$ Decimation occurs without aliasing effect $\rightarrow$ ADC performs decimation and A/D conversion.
- Sometimes, SCF instead of ADC performs filtering and decimation. For either way, control clocks had better be synchronized.

Conventional Postfiltering

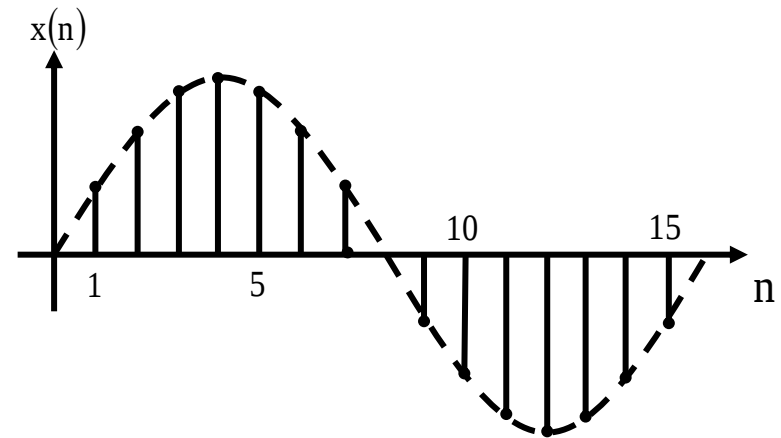


Discrete-time Sinusoidal Signals with Different Sampling Rates

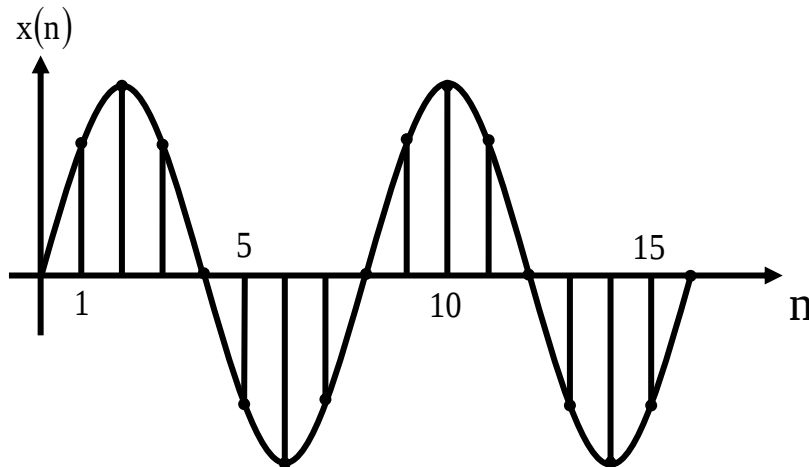
- ∞ samples/cycle



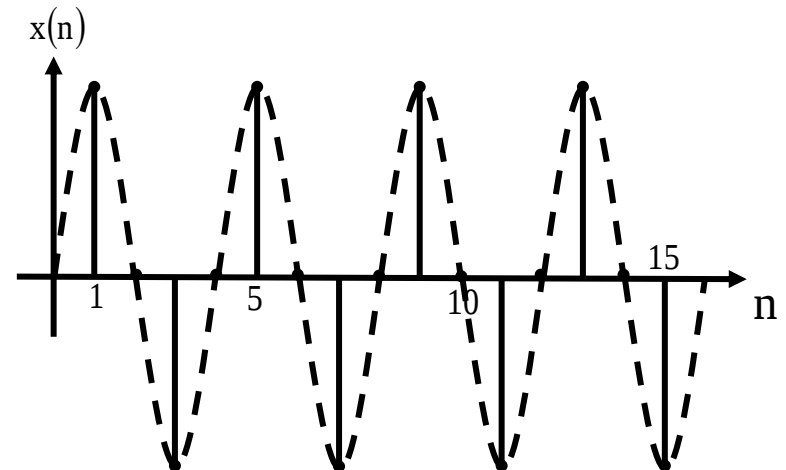
- 16 samples/cycle



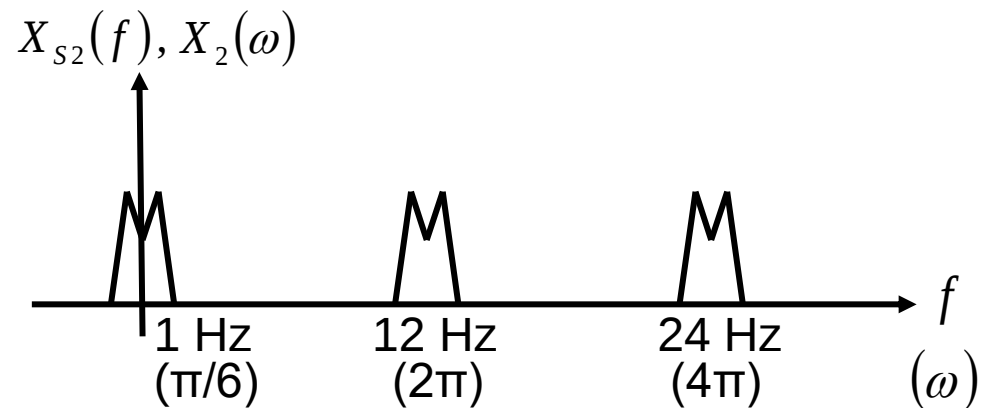
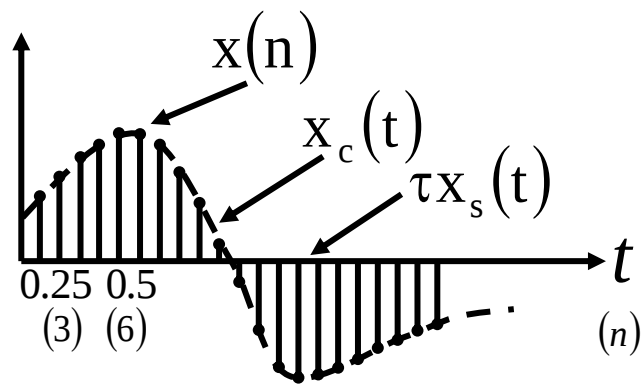
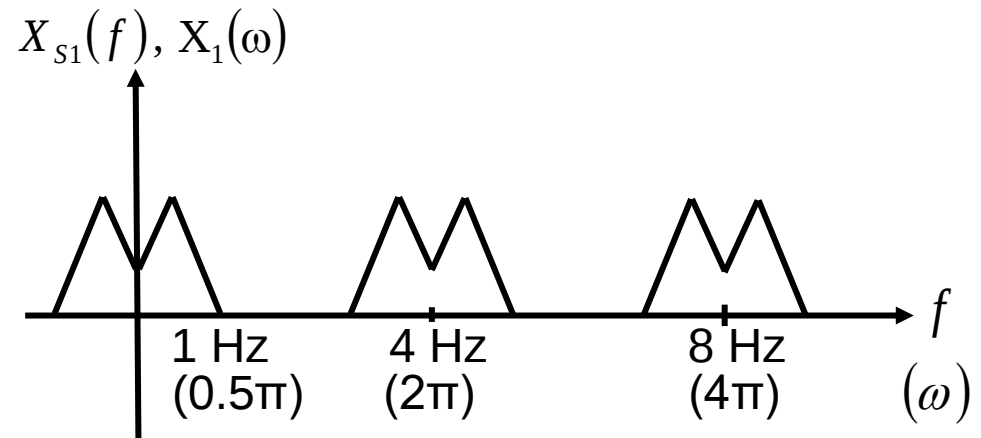
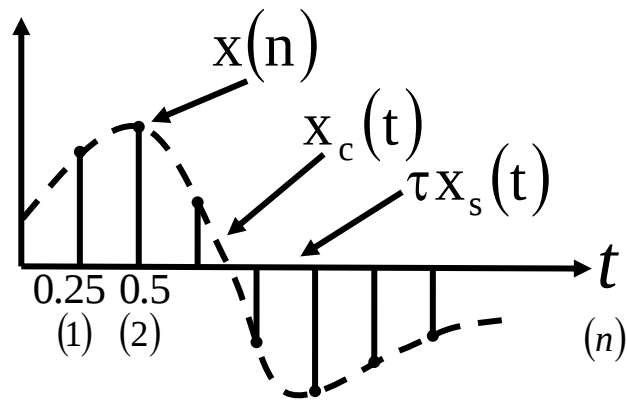
- 8 samples/cycle



- 4 samples/cycle



Time and Frequency Domain of Different Sampling Rates

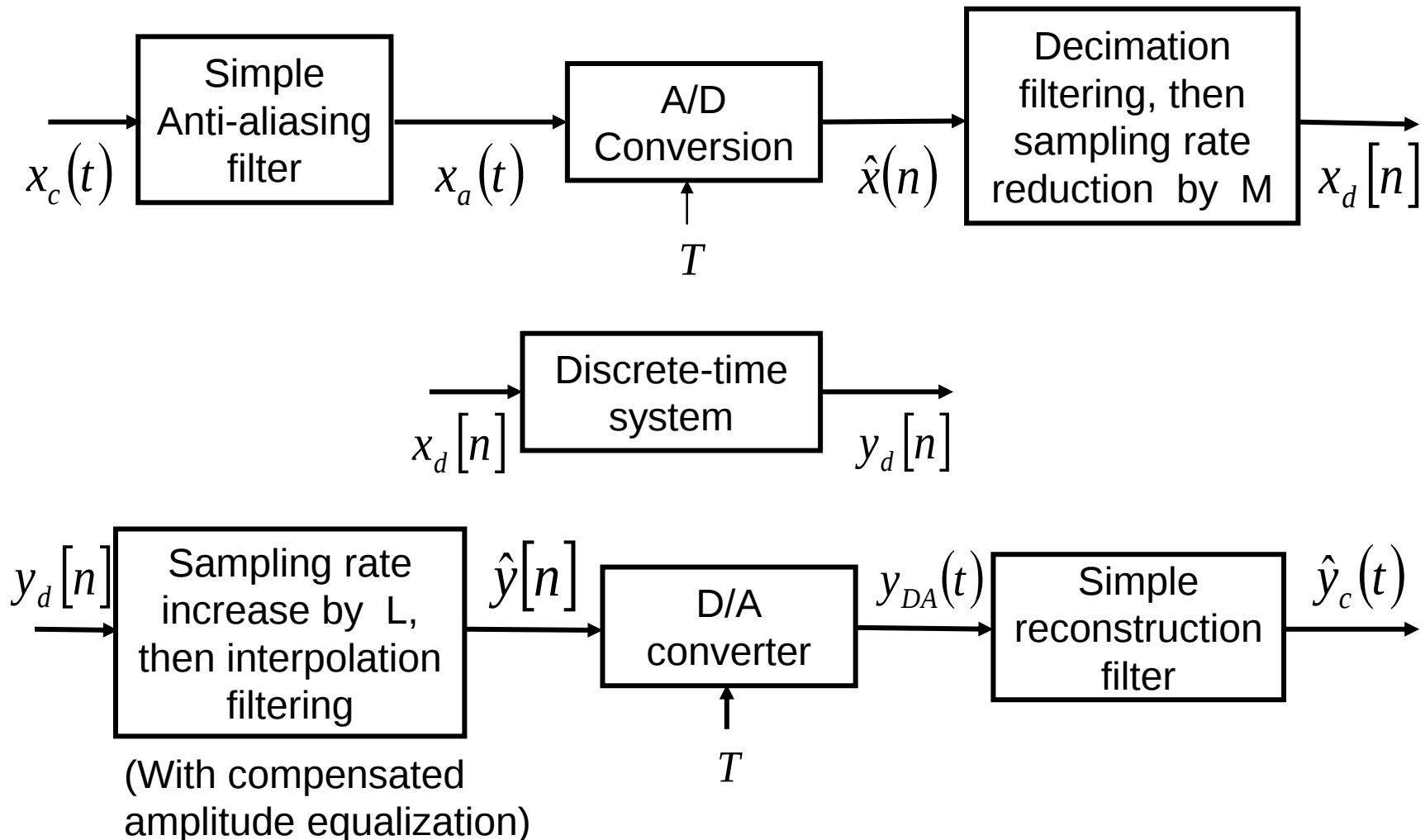


Use of Oversampling Approach to Relax Requirements of Prefilter and Postfilter

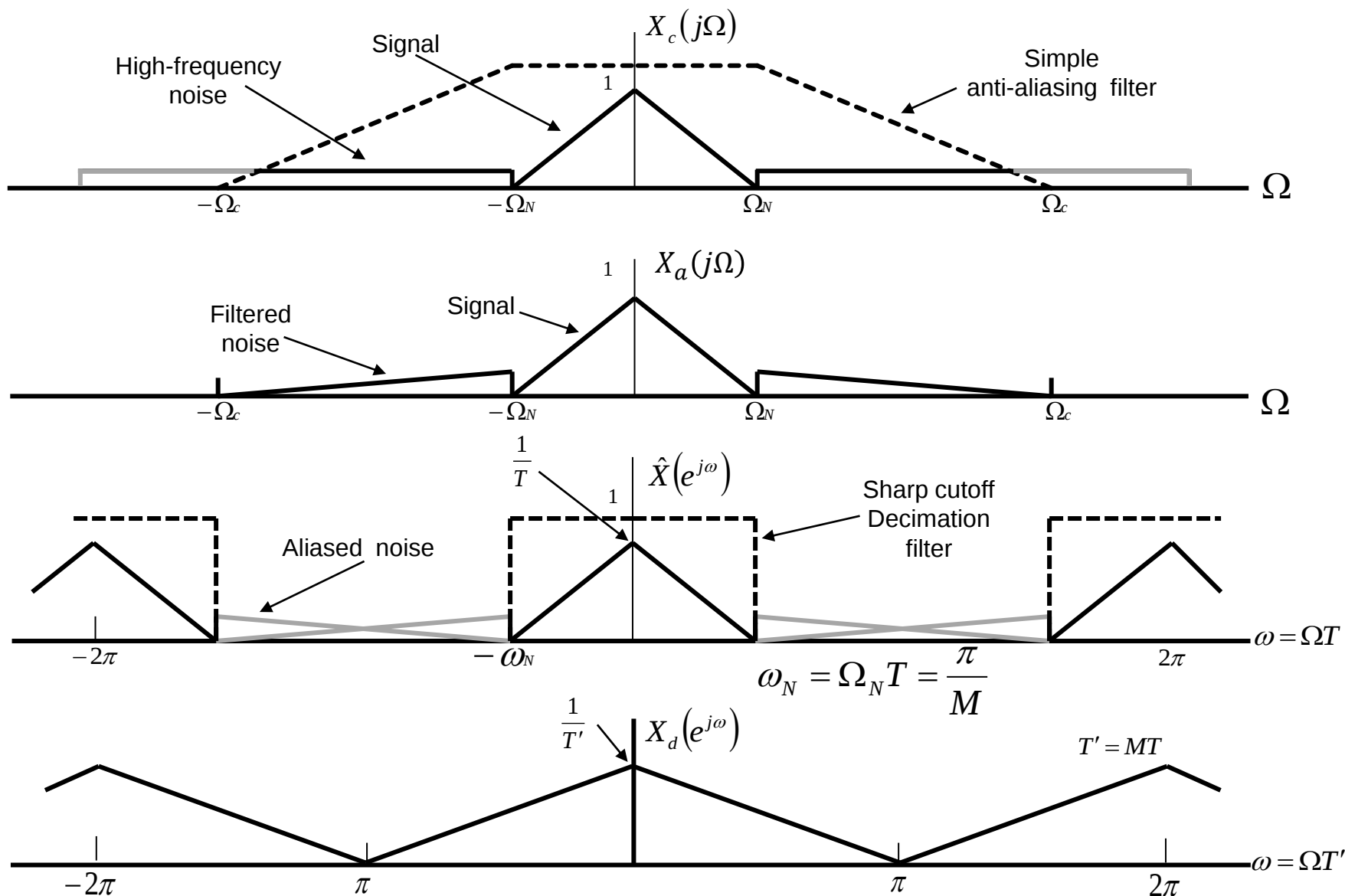
- Front End
 - ◆ Use oversampling A/D converter
 - ◆ Use decimation after A/D conversion
- Back End
 - ◆ Use interpolation before D/A conversion
 - ◆ Use oversampling D/A converter

Use of Oversampling Approach to Relax Requirements of Prefilter and Postfilter (Cont.)

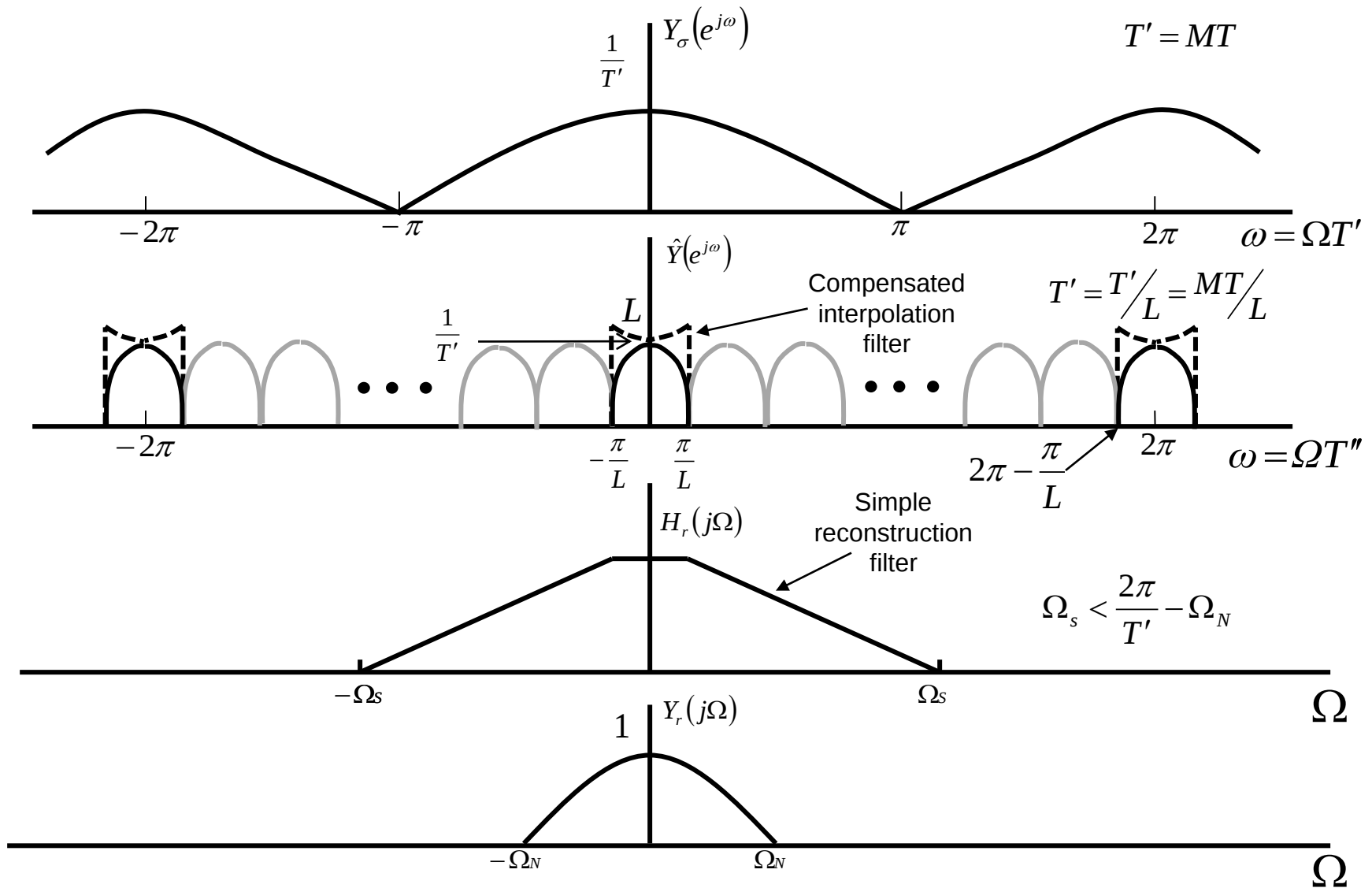
- Example: Block diagram of a signal processing system



Use of Decimation in A/D Conversion

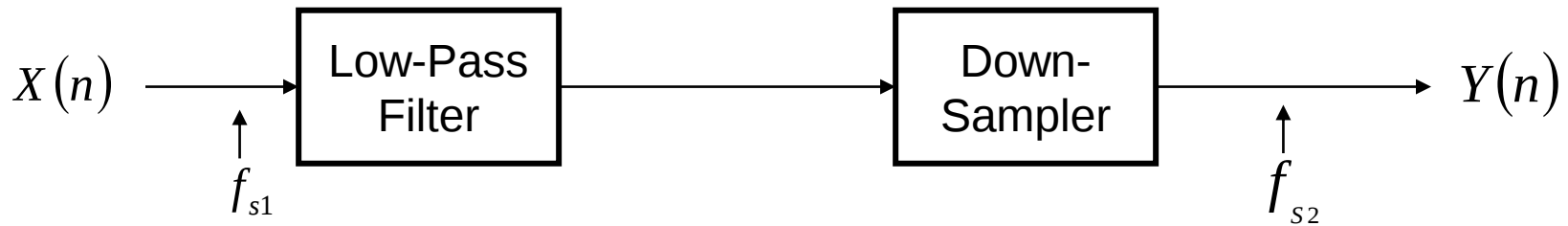


Use of Interpolation in D/A conversion



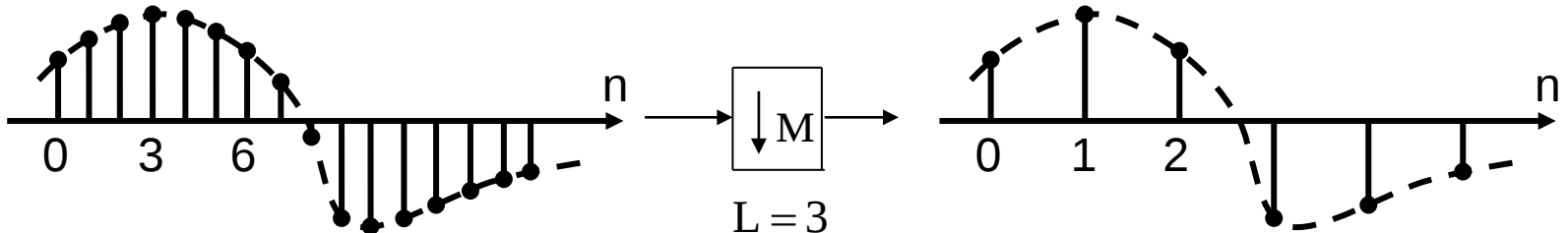
Decimation

- Lowpass filtering + downsampling

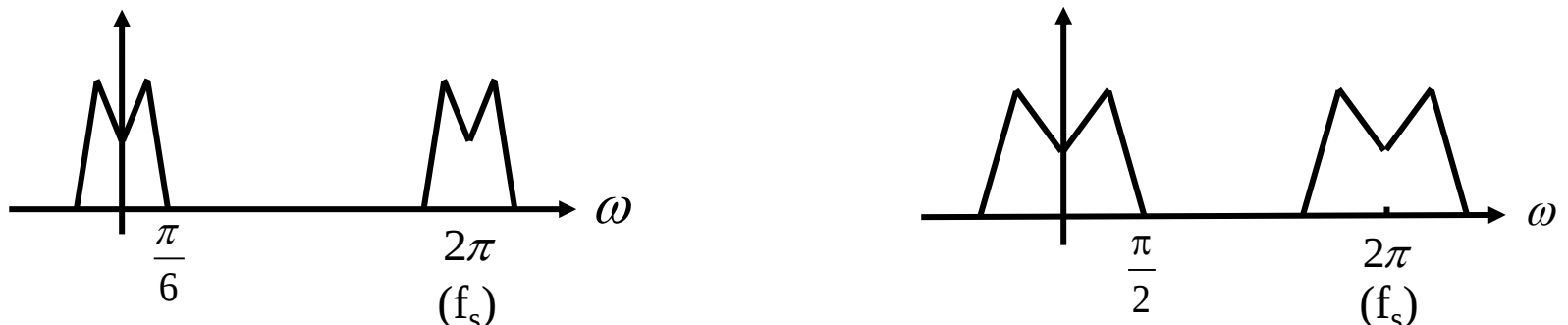


- Downsampling (by 3)

◆ Time-domain



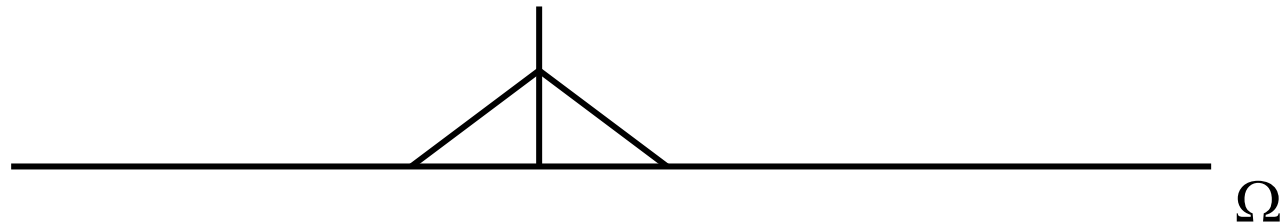
◆ Frequency domain



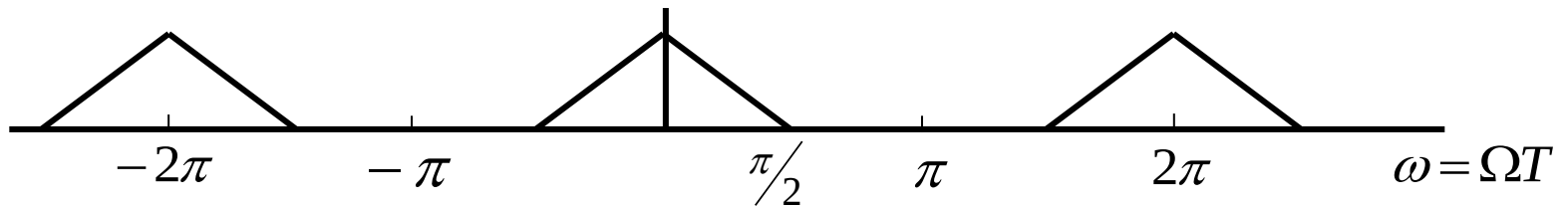
Decimation (Cont.)

- Downsampling with aliasing

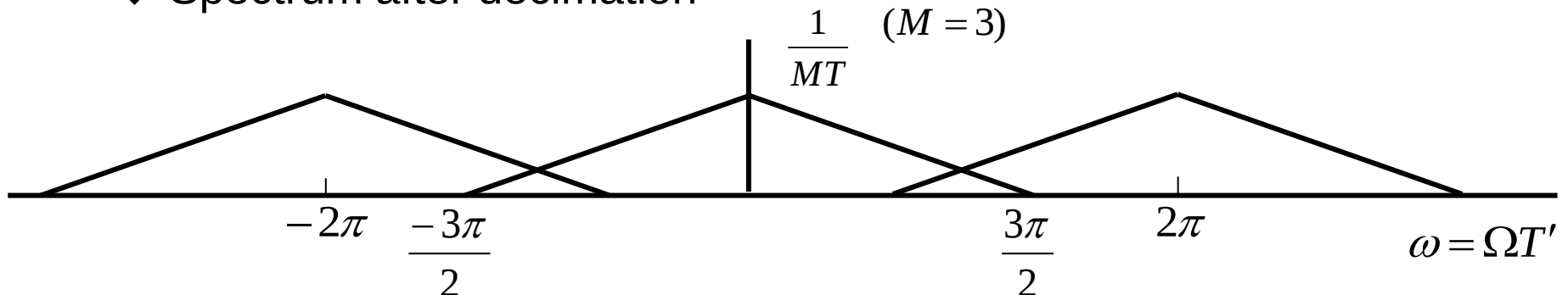
- ◆ Spectrum of original continuous-time signal



- ◆ Spectrum of sampled signal



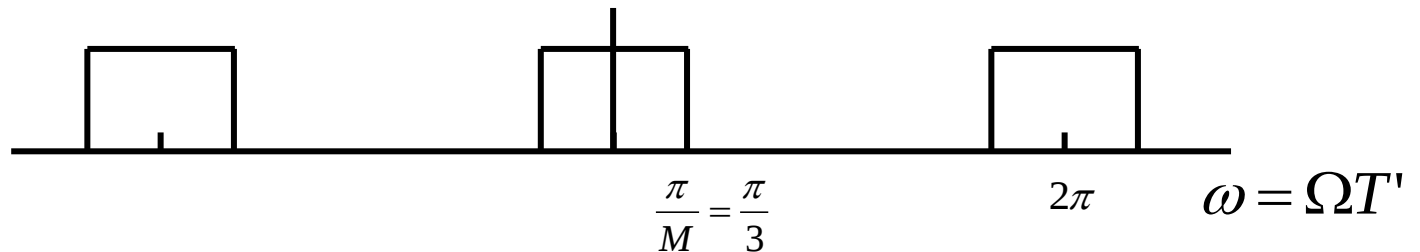
- ◆ Spectrum after decimation



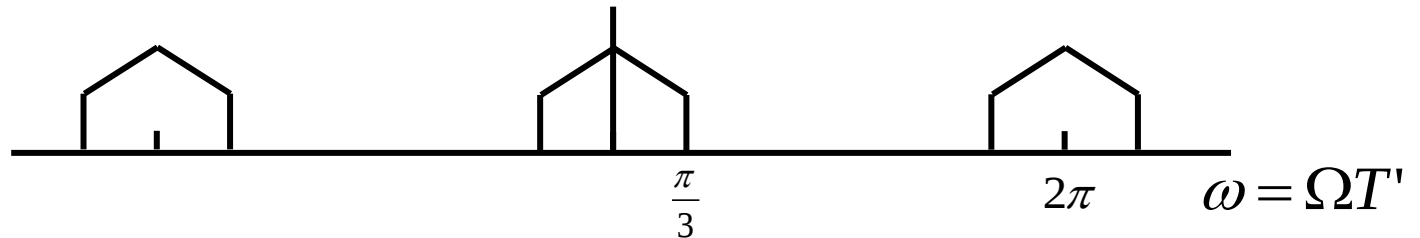
Decimation (Cont.)

- Downsampling with prefiltering to avoid aliasing

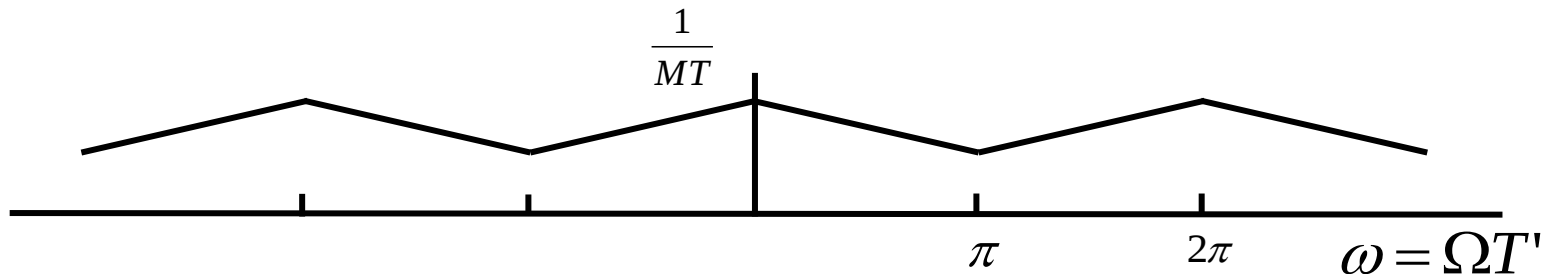
- ◆ Low-pass filtering



- ◆ Spectrum after filtering



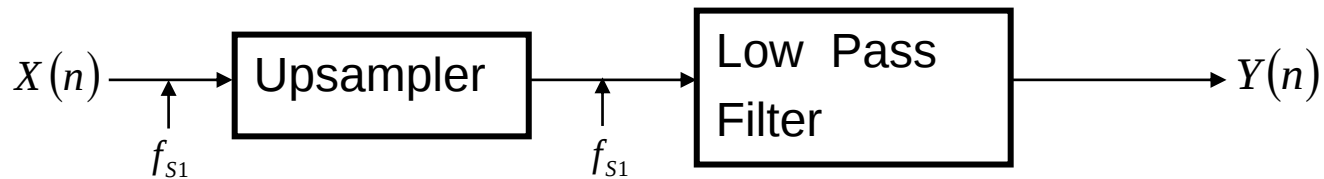
- ◆ Spectrum after decimation



Interpolation

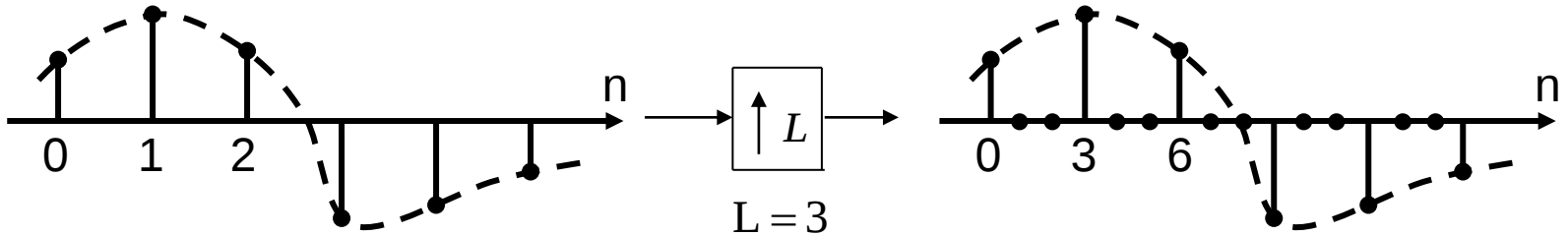
- Upsampling + lowpass filtering

- ◆ Time-domain

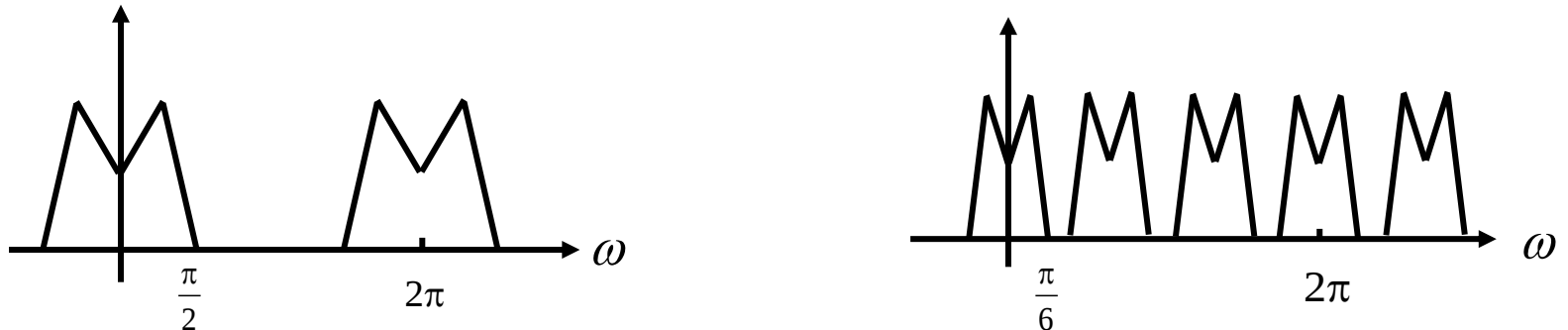


- Upsampling (by 3)

- ◆ Time-domain

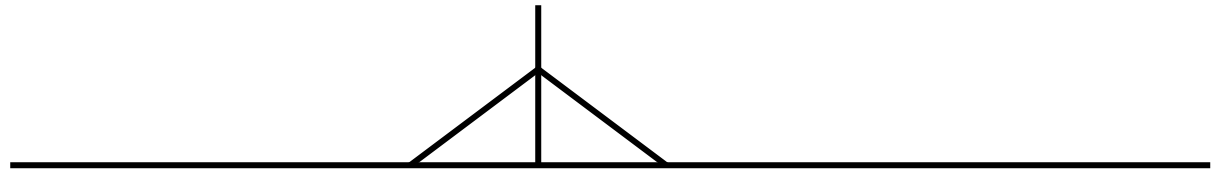


- ◆ Frequency-domain

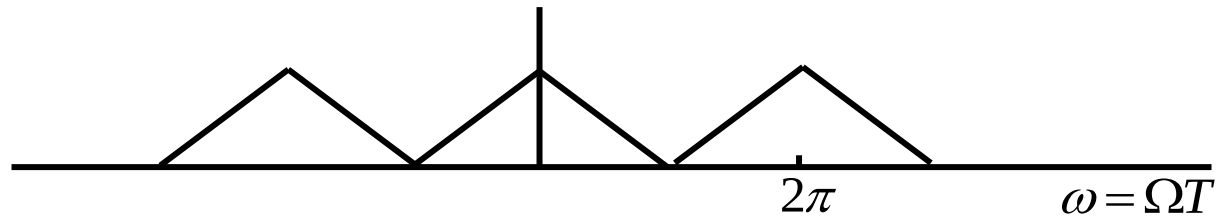


Interpolation (Cont.)

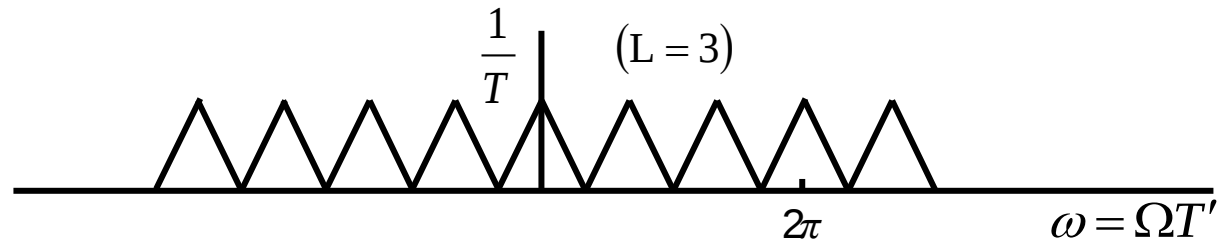
- Baseband signal (Analog)



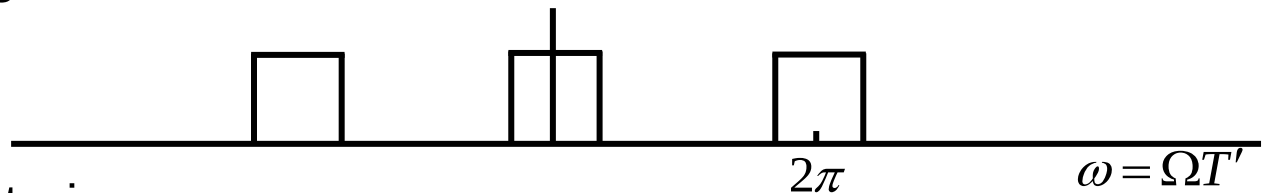
- Original signal (Digital)



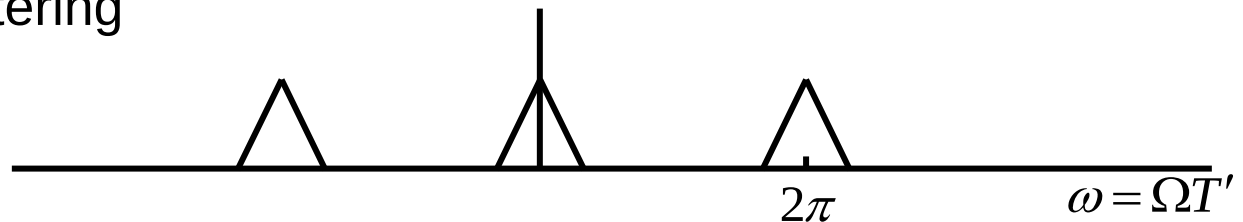
- Upsampling



- Low-pass filtering

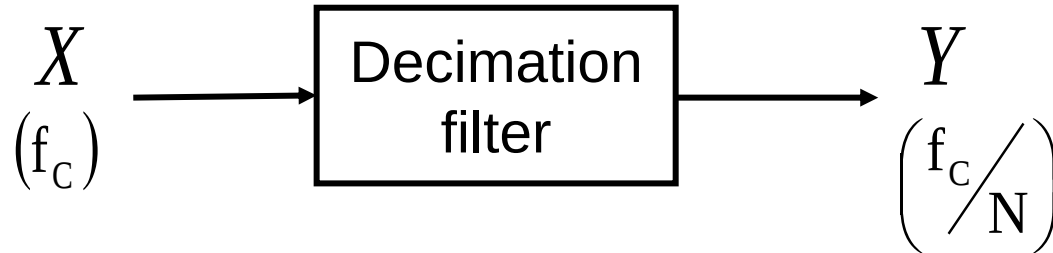


- Spectrum after filtering



Decimate-By-N Filter

- Example: moving average sinc function



Input rate f_c

Output rate $f_c/N = f_d$

$$Y(n) = \frac{1}{N} [X(Nn) + X(Nn - 1) + X(Nn - 2) + \cdots + X(Nn - N + 1)]$$

$$\rightarrow Y(z) = \frac{1}{N} [X(z) + z^{-1}X(z) + \cdots + z^{-(N-1)}X(z)]$$

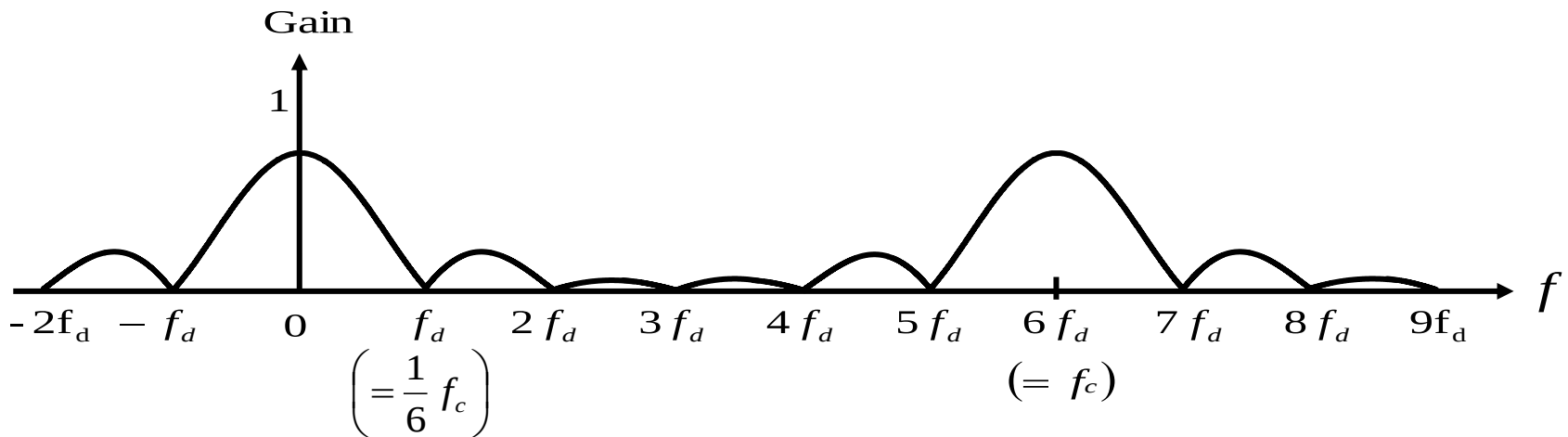
$$\rightarrow \text{Transfer function } \frac{Y(z)}{X(z)} = \frac{1}{N} \left[\frac{1 - z^{-N}}{1 - z^{-1}} \right]$$

Decimate-By-N Filter (Cont.)

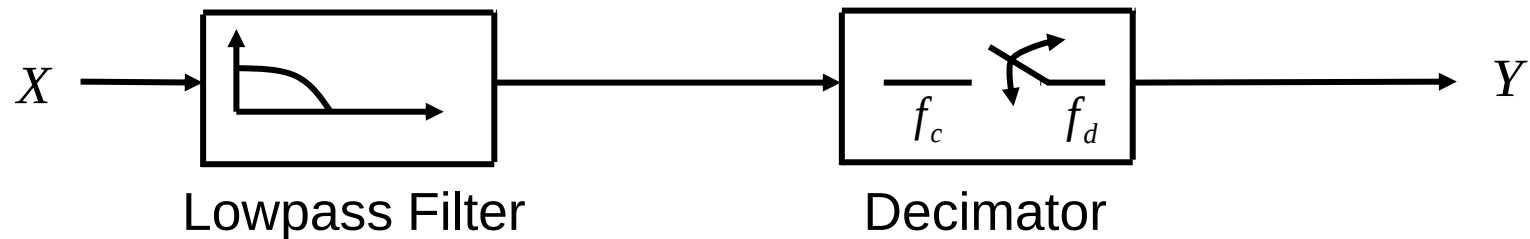
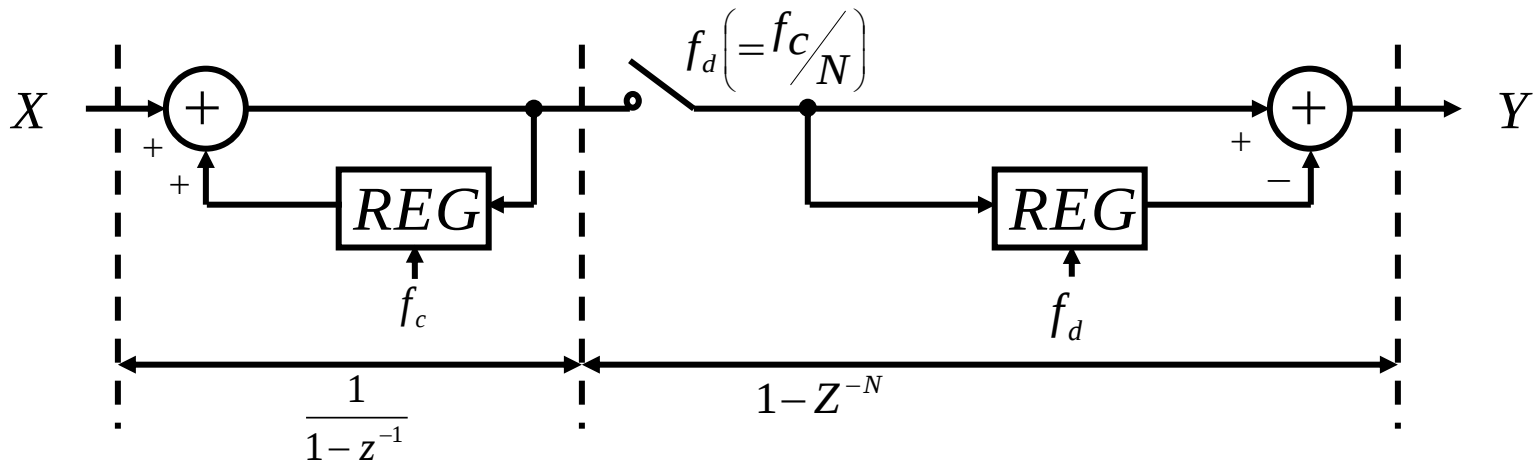
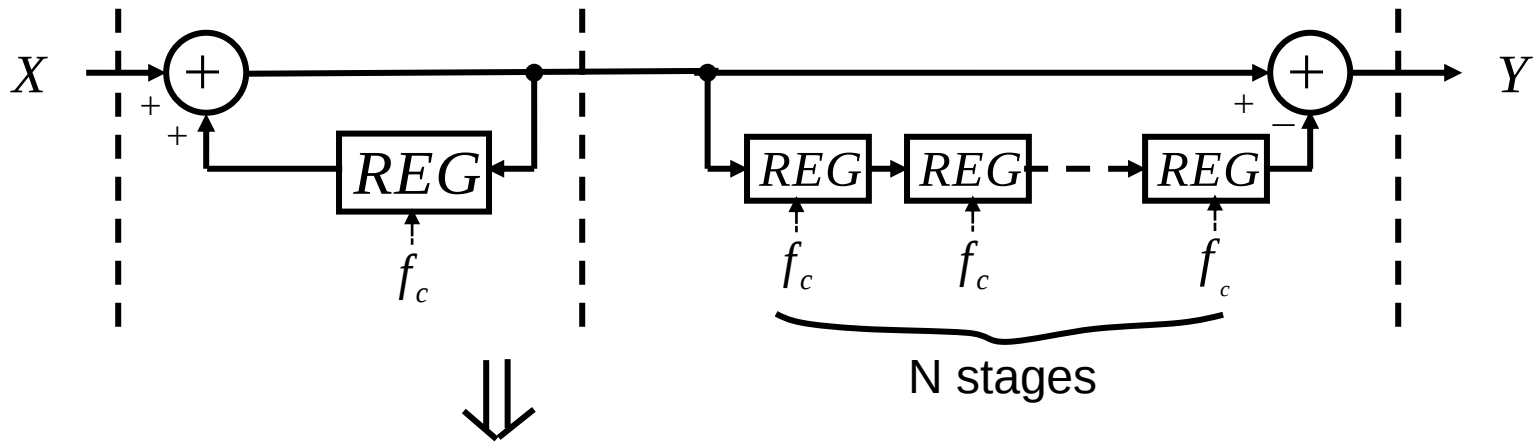
- Gain of sinc filter

$$\frac{1}{N} \left| \frac{1 - z^{-N}}{1 - z^{-1}} \right| = \frac{1}{N} \left| \frac{\sin \frac{N\omega\tau}{2}}{\sin \frac{\omega\tau}{2}} \right| = \left| \frac{\text{sinc} \frac{N\omega\tau}{2}}{\text{sinc} \frac{\omega\tau}{2}} \right| ; \quad z = e^{j\omega\tau} \quad \text{and} \quad \tau = \frac{1}{f_c}$$
$$\text{sinc} X = \frac{\sin X}{X}$$

- Example: N=6



Digital Decimator with Sinc Filtering

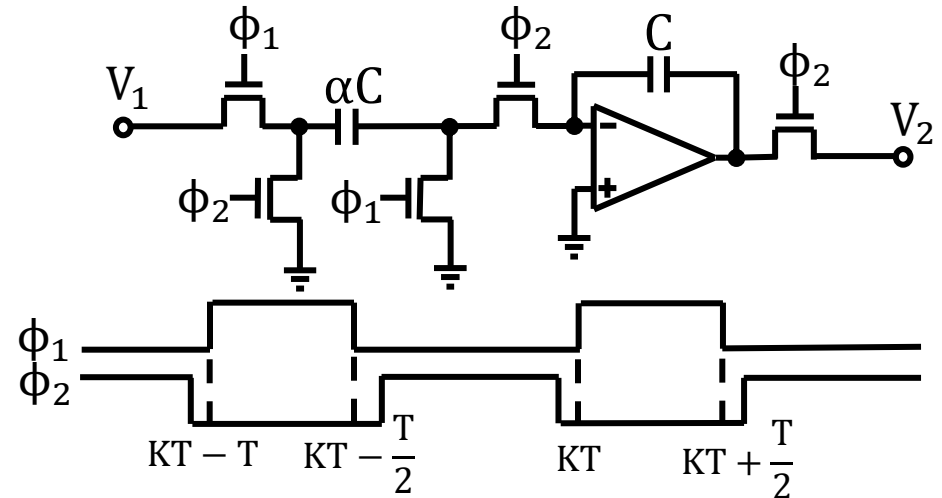


SC Sampling Stage Without Decimation

- Example: Integrator

$$V_2(kT) - V_2(kT - T) = \alpha V_1 \left(kT - \frac{T}{2} \right)$$

$$H_I(z) = \frac{V_2(z)}{V_1(z)} = \frac{\alpha z^{-\frac{1}{2}}}{1 - z^{-1}}$$

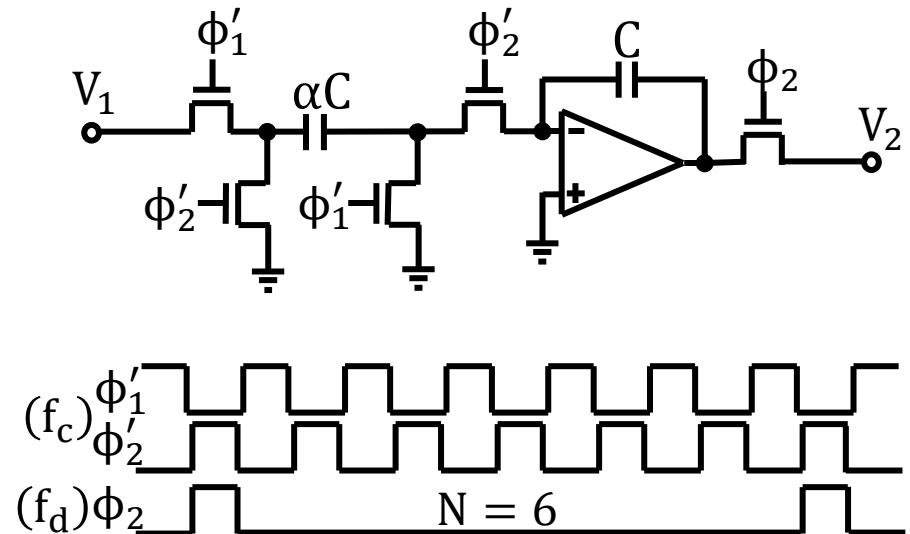
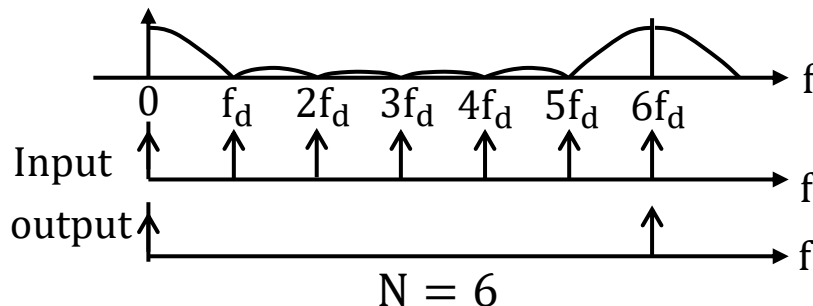


- Example: Modified Integrator

$$H(z) = H_I(z)H_D(z)$$

$H_D(z)$: sinc transfer function

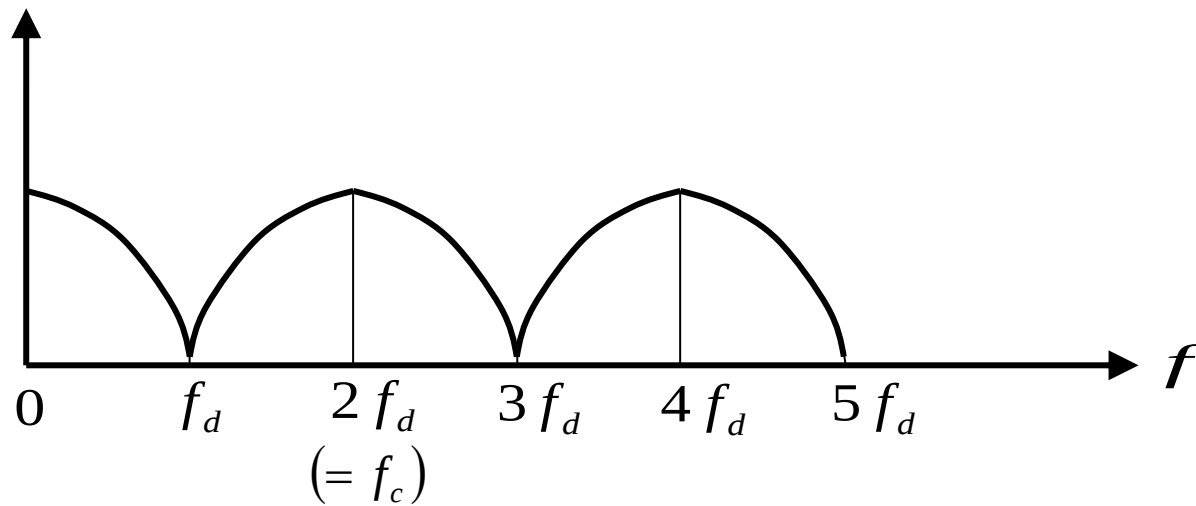
$$|H_D(e^{j\omega\tau})| = \left| \frac{\sin \frac{N\omega\tau}{2}}{\frac{\omega\tau}{2}} \right| ; \tau = \frac{1}{f_c} = T$$



Cosine Filter

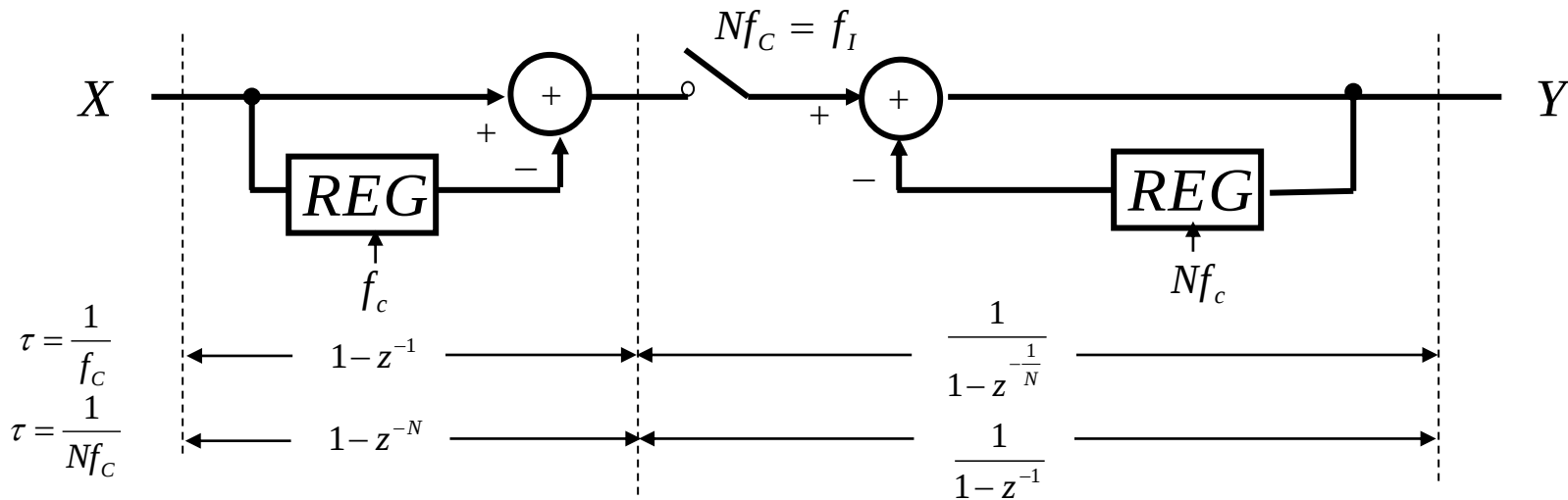
- For $N=2$

$$|H_I(e^{j\omega\tau})| = \left| \frac{\text{sinc } \omega \tau}{\text{sinc } \frac{\omega\tau}{2}} \right| \quad ; \quad \tau = \frac{1}{f_c}$$



Interpolation Filter

- Example: Linear Interpolation (sinc function)

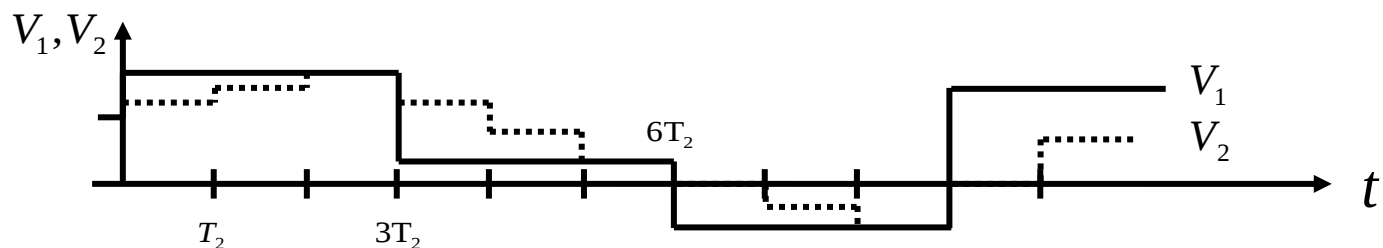
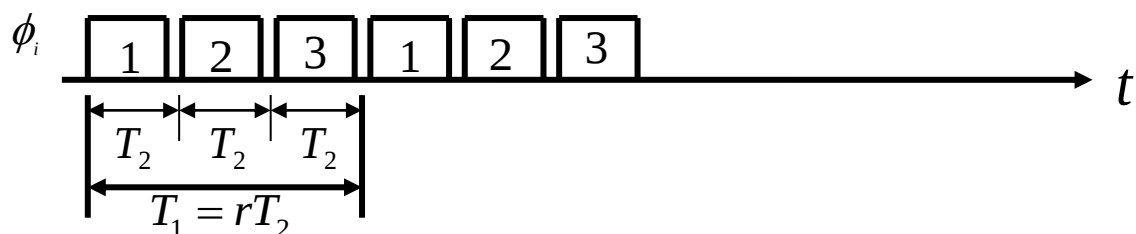
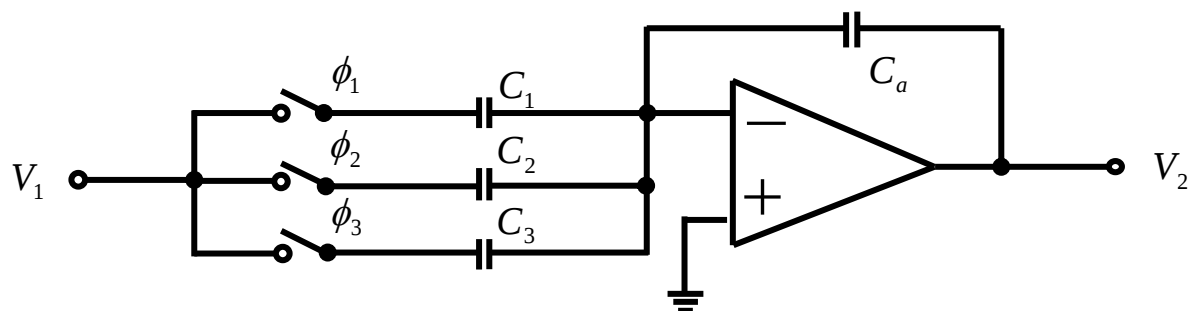


$$H_I(z) = \frac{1}{N} \frac{1 - z^{-N}}{1 - z^{-1}} = \left| \frac{\text{sinc} \frac{N\omega\tau}{2}}{\text{sinc} \frac{\omega\tau}{2}} \right| ; \tau = \frac{1}{f_I}$$

$N = 6$

Magnitude response plot for $N = 6$. The plot shows the magnitude of the transfer function versus frequency f . The main lobe is centered at $f_I = 6f_c$. The side lobes are centered at $f_c, 2f_c, 3f_c, 4f_c, 5f_c, 7f_c$.

SC Sampling Stage With Linear Interpolation



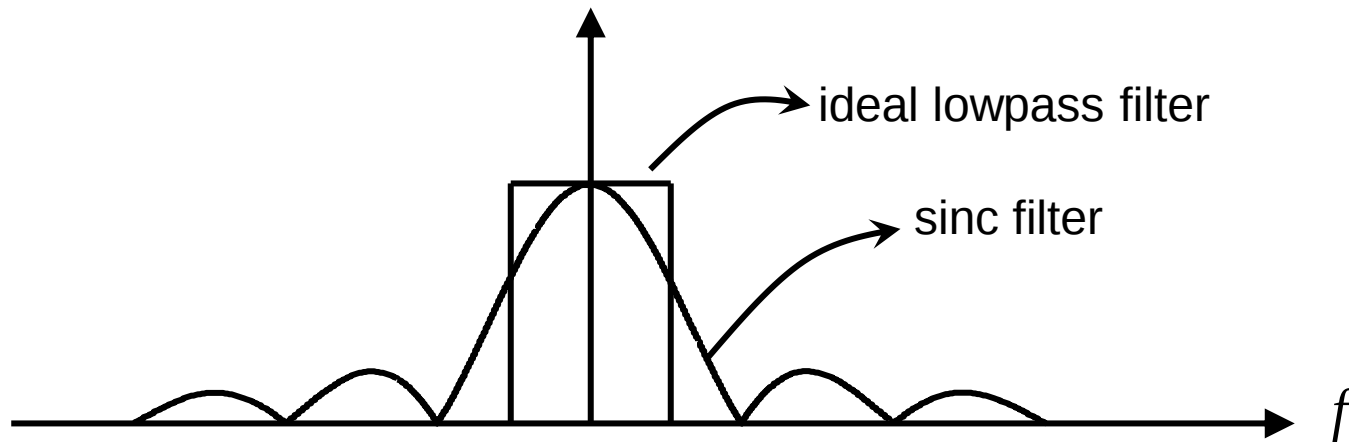
$$V_2(nT_2) - V_2(nT_2 - T_2) = \frac{1}{N} [V_1(nT_2) - V_1(nT_2 - NT_2)]$$

$$H_I(z) = \frac{V_2(z)}{V_1(z)} = \frac{1}{N} \frac{1 - z^{-N}}{1 - z^{-1}} = \frac{1}{N} \sum_{i=0}^{N-1} z^{-i}$$

$$\left| H_I(e^{j\omega\tau}) \right| = \frac{1}{N} \left| \frac{\sin \frac{N\omega\tau}{2}}{\sin \frac{\omega\tau}{2}} \right| ; \quad \tau = \frac{1}{f} = T_2$$

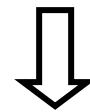
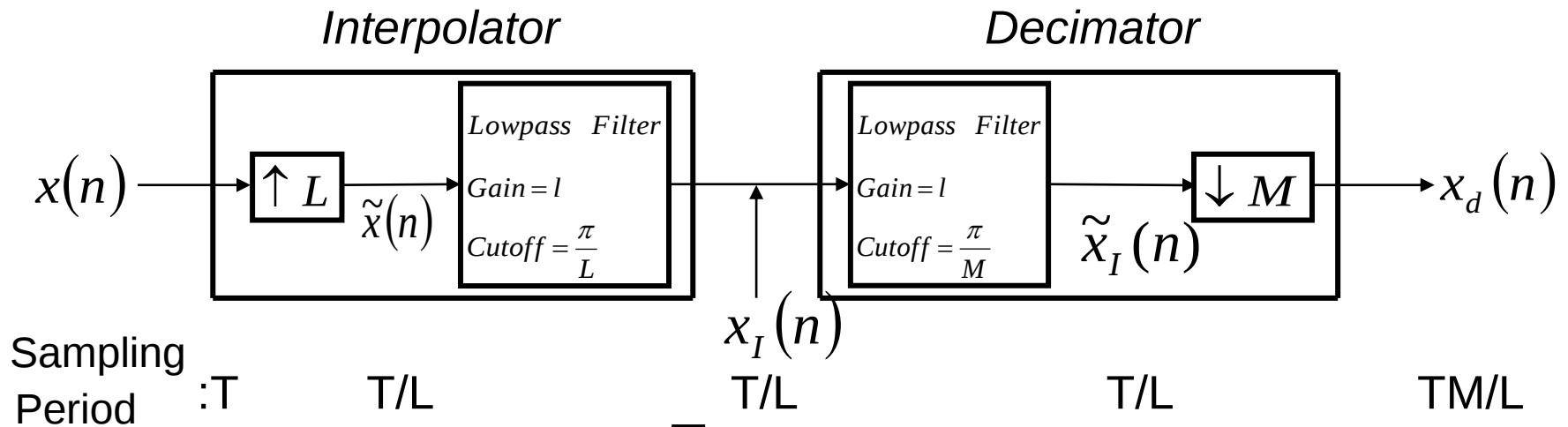
Sinc Filtering Function

- Not ideal lowpass
- FIR (Finite Impulse Response)

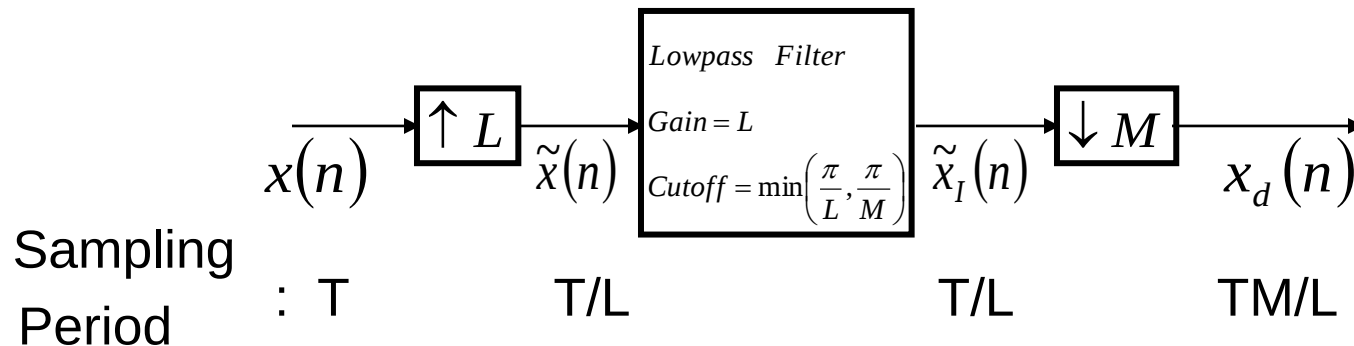


- To implement ideal lowpass function, other approaches can be used.

Changing The Sampling Rate By A Noninteger Factor



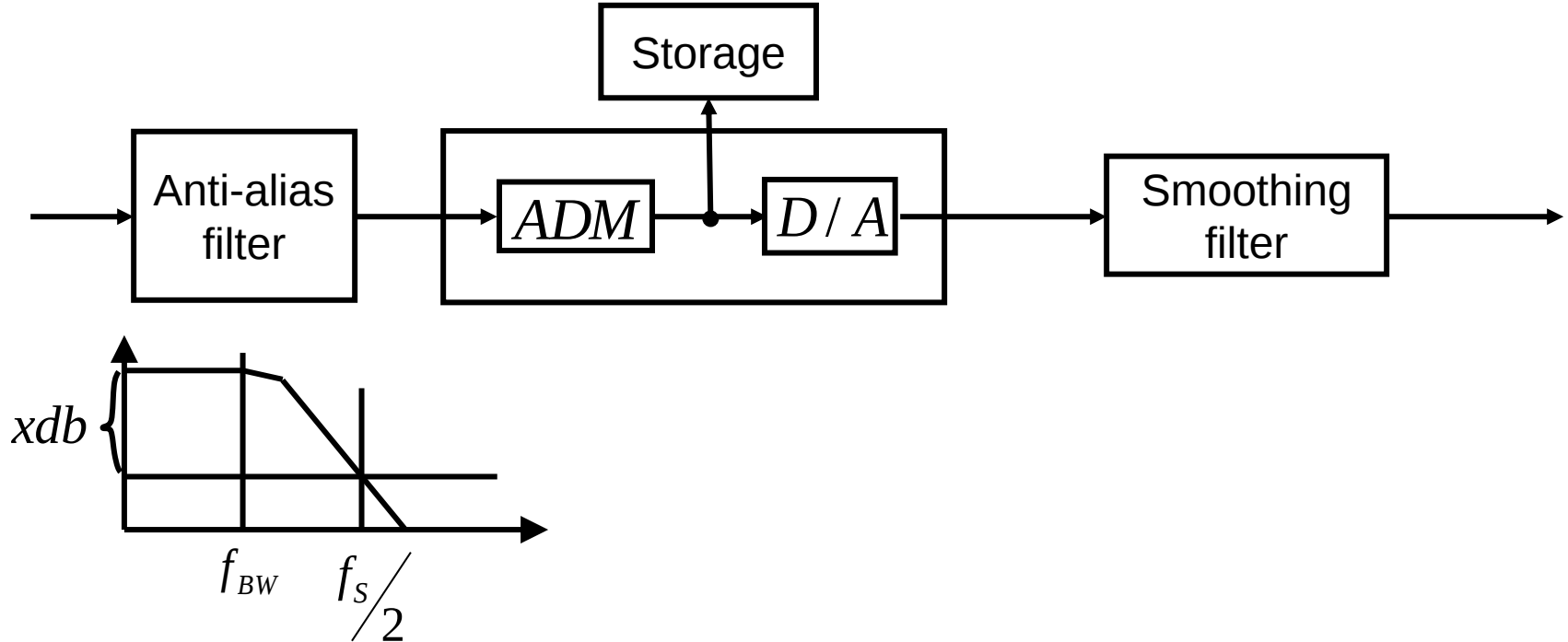
Two lowpass filters
can be combined



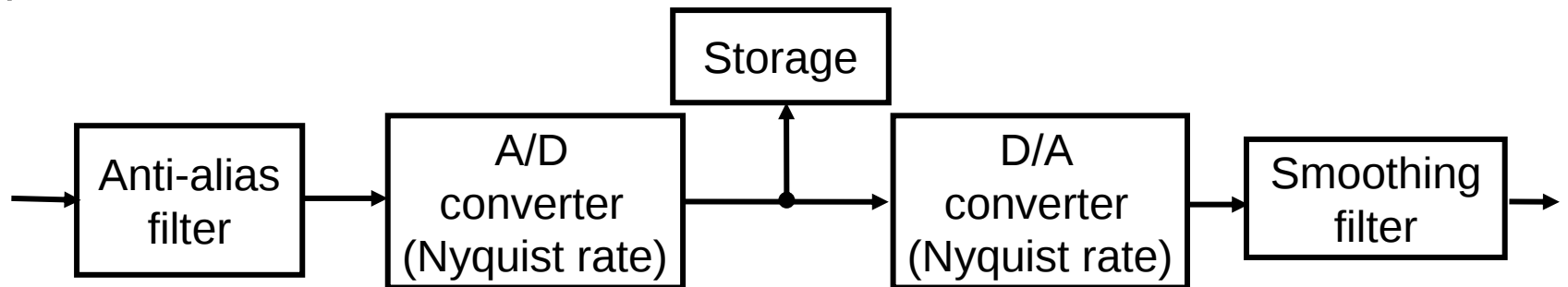
- Noninteger factors can be obtained from properly choosing M and L .

Examples

- Talking Back in the early 1980s



- Speech Reconstruction in the late 1980s



Examples (Cont.)

- Current approach 1:

Better LPF can be used other than sinc one

continuous-time + oversampling + $\underbrace{\text{digital}}_{\text{Decimator and LPF}}$ + DAC + SMF
AAF ADC

◆ Oversampling ADC and digital decimator may be combined.

- Current approach 2:

continuous-time + SCF + conventional + DAC + SMF
AAF ADC
(Nyquist rate)